

**INCAS - National Institute for Aerospace Research “Elie Carafoli”
(under the Aegis of the Romanian Academy)**

**Proceedings of the 37th “Caius Iacob” Conference
on
Fluid Mechanics and its Technical Applications**

16 - 17 November, 2017

Bucharest, Romania

Organizers

**INCAS - National Institute for Aerospace Research
“Elie Carafoli”
(under the Aegis of the Romanian Academy)**

University of Bucharest

**ISMMA - Institute of Mathematical Statistics and
Applied Mathematics of Romanian Academy
“Gheorghe Mihoc - Caius Iacob”**

“Politehnica” University of Bucharest

BUCHAREST

2018

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Universitatea
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The 37th "Caius Iacob" Conference
of
Fluid Mechanics and its Technical Applications

Programme

General Schedule

Thursday 16 November 2017

8.30-10.00	Registration
10.00-10.30	Opening Ceremony <i>In Memoriam Horia Ene</i>
10.30-11.10	Plenary Lectures
11.10-11.30	<i>Coffee Break</i>
11.30-12.50	Communications
12.50-14.00	<i>Lunch</i>
14.00-15.20	Communications
15.20-15.35	<i>Coffee Break</i>
15.35-16.55	Communications
16.55-17.10	<i>Coffee Break</i>
17.10-18.30	Communications
18.30 -	<i>Banquet</i>

Friday 17 November 2017

09.30-10.30	The "Caius Iacob" Prize Award Ceremony, The "Nicolae Tîpei" Prize Award Ceremony
10.30-10.50	<i>Coffee Break</i>
10.50-11.30	Communications
11.30-12.30	Celebrating Excellence
12.30-13.30	<i>Lunch</i>
13.30-15.10	Communications
15.10-	Closing Ceremony

Plenary Lectures

Thursday	Plenary Lectures
“Elie Carafoli” Amphitheatre	
Chairman: Catalin NAE, Alexandru MOREGA	
10.30- 11.10	Horia DUMITRESCU and <i>The Origin of Shear Turbulence</i> Vladimir CARDOS

Thursday	Basic Methods in Fluid Mechanics	
“Elie Carafoli” Amphitheatre		
Chairman: Valentin BUTOESCU, Alexandru DUMITRACHE		
11.30- 11.50	Mihai Victor PRICOP, Mihai Leonida NICULESCU, Marius Gabriel COJOCARU, Andra MANȚIGHIAN, Ionuț BUNESCU	<i>Low fidelity models based airfoil optimization</i>
11.50- 12.10	Viorel ANGHEL	<i>Integral Method for Static, Dynamic, Stability and Aeroelastic Analysis of Beam Like Structure Configurations</i>
12.10- 12.30	Radu BOGATEANU, Vladimir CARDOS, Horia DUMITRESCU	<i>The original sin of the paradigms in turbulence</i>
12.30- 12.50	Ana-Maria BORDEI, Andrei HALANAY	<i>Stability analysis for an UAV model in a longitudinal flight</i>
Chairman: Viorel Anghel, Radu BOGATEANU		
14.00- 14.20	Valentin BUTOESCU	<i>Wing in Ground Effect over a Wavy Surface</i>
14.20- 14.40	Adrian-Nicolae BUȚURACHE, Grigore CICAN	<i>Design, Manufacturing and Testing of Hybrid Rocket Engines for Aerospace Propulsion</i>
14.40- 15.00	Alexandru DUMITRACHE, Florin FRUNZULICA, Octavian PREOTU, Tudor IONESCU	<i>Thrust and Jet Directional Control Using the Coandă Effect</i>
15.00- 15.20	Mihai Leonida NICULESCU, Mihai Victor PRICOP, Marius Gabriel COJOCARU, Maria Cristina FADGYAS	<i>CFD Based Wind Tunnel Solid Wall Corrections for 2D High Lift Configurations</i>

Thursday	Equations of Mathematical Physics	
“Nicolae Tîpei” Amphitheatre		
Chairman: Ruxandra STAVRE, Gelu PAŞA		
11.30- 11.50	Sanda CLEJA-ȚIGOIU, Victor ȚIGOIU	<i>Continuum models for crystalline elasto-plastic materials with microstructural defects</i>
11.50- 12.10	Alin Alexandru DOBRE, Alexandru Mihail MOREGA, Mihaela MOREGA	<i>Probing the Relevance of the Cardiovascular Impedance as a Localized Investigation Technique - A Numerical Experiment</i>
12.10- 12.30	Daniela ENCIU, Andrei HALANAY, Ioan URSU	<i>A Predictive Type Approach of Control Delay for Electrohydraulic Servomechanism</i>
12.30- 12.50	Cecil P. GRÜNFELD	<i>On a Class of Nonlinear Evolution Equations for Positive Self-Adjoint Trace-Class Operators</i>
Chairman: Sanda CLEJA-ȚIGOIU, Dan POLIȘEVSKI		
14.00- 14.20	Ștefan HOTHAZIE	<i>On a Boost in Computational Resource Utilization of the Finite Difference Scheme</i>
14.20- 14.40	Stelian ION, Dorin MARINESCU, Stefan Gicu CRUCEANU	<i>Analytical and numerical solution of Riemann Problem for a class of discontinuous hyperbolic systems</i>
14.40- 15.00	Gelu PAŞA	<i>Non-Newtonian effects in three-layer Hele-Shaw displacements</i>
15.00- 15.20	Claudiu PĂTRAȘCU, Eugen CHIRIAC, Corneliu BĂLAN	<i>Drop dispensing in a viscous outer liquid</i>

Thursday	Dynamical Systems	
“Nicolae Tîpei” Amphitheatre		
Chairman: Victor ȚIGOIU, Cecil Pomiliu GRÜNFELD		
15.35- 15.55	Isabelle GRUAIS, Dan POLIȘEVSKI	<i>Model of Two-Temperature Convective Transfer in Porous Media</i>
15.55- 16.15	Dumitru POPESCU	<i>Effect of the external medium of finite size on the swelling stage of a pulsatory liposome</i>
16.15- 16.35	Ruxandra STAVRE	<i>Asymptotic analysis for a general case of fluid-structure interaction model</i>
16.35- 16.55	Gabriela STROE, Irina-Carmen ANDREI	<i>Study about ATS surveillance systems in the Flight Information Service</i>
Chairman: Corneliu BĂLAN, Ștefan-Gicu CRUCEANU		
17.10- 17.30	Alina-Ioana CHIRA, Florin COSTACHE, Achim IONITA	<i>Dynamic model of a hybrid platform, Tandem Quadrorotor X with aerodynamic effects</i>
17.30- 17.50	Florin COSTACHE, Mihaela-Luminita COSTEA, Achim IONITA	<i>Modelling and Simulation of jet transport motion in terminal phases</i>
17.50- 18.10	Gabriela STROE, Irina-Carmen ANDREI	<i>Integrated Air Traffic Management Systems</i>
18.10- 18.30	Serena Cristiana VOICU, Mirela-Madalina BIVOLARU, Adrian Mihail STOICA	<i>Sliding Mode Control of a Hexacopter</i>

Thursday	Mathematical Modeling	
“Elie Carafoli” Amphitheatre		
Chairman: Florin FRUNZULICĂ, Marius Gabriel COJOCARU		
15.35- 15.55	Mihai Leonida NICULESCU, Maria Cristina FADGYAS, Marius Gabriel COJOCARU, Mihai Victor PRICOP, Mihaita Gilbert STOICAN, Dumitru PEPELEA	<i>Aerodynamics and Chemistry for an Earth Reentry Capsule</i>
15.55- 16.15	Corneliu BERBENTE, Sorin BERBENTE	<i>Possible Simple Structures of the Actual Universe to Include General Relativity Effects</i>
16.15- 16.35	Grigore CICAN, Ionuț-Florian POPA, Ana-Maria T. ANDREESCU	<i>Magnetic Nozzles for space plasma thrusters</i>
16.35- 16.55	Mihnea GALL, Vlad Alexandru POPA	<i>Numerical investigation of vertical axis wind turbine based on lift-drag forces</i>
Chairman: Mihail Leonida NICULESCU, Corneliu BERBENTE		
17.10- 17.30	Grigore CICAN, Florin FRUNZULICĂ, Marius BREBENEL, Luminita DRAGASANU, Marius DEACONU	<i>Approach to determine the noise levels inside helicopter cabin. Case study to obtain input data for helicopter cabin noise reduction campaign</i>
17.30- 17.50	Alexandru-Iulian ONEL, Tudorel-Petronel AFILIPOAE, Ana-Maria NECULĂESCU, Mihai-Victor PRICOP	<i>Mathematical model for fast drag coefficient estimations with application to small launcher optimisation</i>
17.50- 18.10	Adrian STOICA	<i>Incompressible flow past elliptic ring wings of high aspect ratio</i>

Friday	Technical Applications	
“Elie Carafoli” Amphitheatre		
Chairman: Tiberiu SALAORU, Stelian ION		
10.50- 11.10	Irina-Carmen ANDREI	<i>Design Issues in Case of Pumps for Liquid Propelled Rocket Engines</i>
11.10- 11.30	Marius Alexandru PANAIT	<i>Supersonic and transonic Mach probe for calibration control in the Trisonic Wind Tunnel</i>
Chairman: Irina-Carmen ANDREI, Mihai-Victor PRICOP		
13.30- 13.50	Ana-Maria NECULĂESCU, Tudorel-Petronel AFILIPOAE, Alexandru-Iulian ONEL, Alexandru-Gabriel PERȘINARU, Alexandru-Mihai CIȘMILIANU, Ionuț-Cosmin ONCESCU, Mihaela NĂSTASE, Camelia-Elena MUNTEANU, Adrian TOADER, Mihai-Victor PRICOP	<i>Micro-launch vehicles studies</i>
13.50- 14.10	Alexandru-Gabriel PERȘINARU, Alexandru-Mihai CIȘMILIANU, Alexandru MARIN, Adrian TOADER, Alexandru BURCĂ, Ciprian CHIVU, Costin VIȘOIU, Ionuț-Cosmin ONCESCU, Mihaela NĂSTASE, Camelia-Elena MUNTEANU, Ana-Maria NECULĂESCU, Mihai-Victor PRICOP	<i>Demonstrator for Technologies Validation</i>
14.10- 14.30	Tiberiu Adrian SALAORU, Irina Carmen ANDREI	<i>Fuel level monitoring system by using mechanical guided waves, with application to a mixed flows turbofan engine</i>
14.30- 14.50	Tiberiu Adrian SALAORU, Irina Carmen ANDREI	<i>Fuel consumption monitoring based on the tank gas pressure variation method, with application to a mixed flows turbofan engine</i>
14.50- 15.10	Angi NORBERT, Angel HUMINIC, Csaba ANTONYA	<i>Flight Control System Design and Analysis of a Light Sport Aircraft with Emphasis on Multybody Dynamics and Aerodynamic Analysis</i>

Book of Abstracts

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Note:

- Please, note that the responsibility for the accuracy of expression in English belongs to the authors.

Plenary Lectures

New Methodology for Aircraft Modelling for Descent at Constant CAS/Mach and Descent Rate

Ruxandra Mihaela BOTEZ

Abstract: A new method is presented for the computation of performance databases used in the Flight Management System of commercial aircrafts. This method uses a mathematical model built for the Cessna Citation X business aircraft. This model was developed at the Laboratory of Applied Research in Active Controls, Avionics and AeroServoElasticity LARCASE by use of a Level D research flight simulator for the Cessna Citation X. The level D is the highest level given for the flight dynamics certification. The descent was performed with a constant calibrated airspeed or Mach number. Firstly, the variation of weight due to fuel consumption was not considered, thus the outputs of our performance database were the true airspeed, flight path angle, angle of attack and net thrust. The mathematical model was obtained from the equations of motion, thus from the application of the Newton's second law applied to a powered flight. The aircraft data required in order to complete the mathematical model are the geometry of the airplane, the aerodynamic database and the engine database. The tests are performed for a large set of configurations of weights and canters of gravity positions, and different CAS/Mach and vertical speeds for each configuration. The results show that for the majority of the tests, the model can predict the evolution of the output parameters with an accuracy of 5%. Most of the time, the overpassing concerns the calculation of the net thrust.

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The Origin of Shear Turbulence

Horia DUMITRESCU, Vladimir CARDOS*

Abstract: Previously, the turbulent flows were studied as they have been disregarding their origin, and without looking into some of the details of the mechanism of turbulence production and sustainment. This approach focusing its attention particularly on the details of the fluctuating motion superimposed on the main motion led to most of controversial and misunderstood results from which the law of equal action and reaction, and the circulation-conserving were excluded. The paper is aiming at removing these drawbacks and presenting the turbulence phenomenon as a mechanical process triggered off at the beginning of motion. The fluid as deformable continuum without a definite shape must be guided by some physical surfaces, where the rotor-translational motion approximation is more suited than the no-acceleration parallel flow approximation. The main features of shear turbulence are described by means the mechanical prototype of a perturbed rotor-translational motion continuously self-accelerating at wall and conserving the mass and angular momentum.

Key Words: Laminar-turbulent transition, Shear turbulence, Rotor-translational flow model

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Section 1. Basic Methods in Fluid Mechanics

Rotor Vibrations Study in Non-Inertial Frame

Violeta-Andreea ANASTASE*¹, Ion STROE²

Abstract: This paper treats the problem of the elastic structures like the behaviour of jet engines in non-inertial frames. The main case of a flexible rotor shaft system with damped and undamped motion is analyzed by dynamic point of view, based on the approximate methods. The two degrees of freedom model, considering the expression of the system kinetic and strain energies, is based on the Lagrange equation

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_k} \right) - \frac{\partial L}{\partial q_k} = 0, \text{ in which the torque vibrations are neglected. In this equation, the Lagrange function } L$$

represents the sum between the kinetic energy E and the force function U . The eigenvalues problem is solved by developing a MATLAB code for the translation and rotation particular motion, revealing the natural frequencies and the modal shapes. Consequently, the Campbell diagram (taking into account the natural frequencies as a function of rotation speed) is plotted in order to show the relationship between the rotating machinery movement in inertial and non-inertial reference systems. Also, the diagrams of x coordinate variation as a function of time highlights the vibration amplitude of the rotor shaft system in the moment of reaching the critical speed spectrum.

Key Words: Rotor, shaft, disk, dynamics, vibration

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Integral Method for Static, Dynamic, Stability and Aeroelastic Analysis of Beam Like Structure Configurations

Viorel ANGHEL

Abstract: This work presents a synthesis of the use of an integral approximate method based on structural influence functions (Green’s functions) concerning the behaviour of beam like structures. The domains of the use of this integral method are static, dynamic, stability and aeroelasticity. The integral method starts from the differential equations governing the bending or/and torsional behaviour of a beam. These equations are put in integral form by using appropriate Green’s functions, according to the boundary conditions. Choosing a number of n collocation points on the beam axis, each integral are then computed by a summation using weighting numbers. This approach is suitable for conventional Euler-Bernoulli beams and also for the thin-walled open or closed cross-section beams which can have bending-torsion coupling. Generally, for a static analysis this approach leads to a linear system of equations (the case of the lift aeroelastic distribution analysis) or to an eigenvalues and eigenvectors problem in the case of dynamic, stability or divergence analysis.

Key Words: Integral Method, Green Functions, Collocation, Static, Dynamic, Stability, Aeroelasticity

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The original sin of the paradigms in turbulence

Radu BOGATEANU^{*1}, Vladimir CARDOS², Horia DUMITRESCU²

Abstract: It is very important to make a clear distinction between physical phenomena and the mathematical problem of turbulence, i.e. between the observable experiences and the theories attempting to explain them. Indeed, unlike other complicated phenomena, turbulence is easily observed, but is extremely difficult to interpret understand and explain. Thus, though there exist a number of beautiful collections of images of turbulent flow, the ignorance of the nature of turbulence gave birth to branchy phenomenology of turbulence creating smaller or bigger mathematical “monsters” from simple semi-empirical approaches up to strange attractors. A lot of these approaches are of unknown validity and obscured physical and mathematical problem of turbulence. The physical/ mathematical problems of turbulence being ones of paradigmatic nature concerning the behaviour of flows close to boundaries, in this paper we propose a different approach of the fluid-boundary contact problem, i.e. a new law for the wall-bounded flows, complying better with the action-reaction law responsible for most controversial aspects of turbulence.

Key Words: Navier-Stokes Equations, Stokes’ hypothesis, wall-bounded law

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Stability analysis for an UAV model in a longitudinal flight

Ana-Maria BORDEI* and Andrei HALANAY

Abstract: In this paper we present the stability analysis of the equilibria in a longitudinal flight of an unmanned aircraft with constant forward velocity. The motion of the aircraft is described using delay differential equations with constant delays, the delay being considered in flight control compartment. The goal is to study the effects of the delays for the stability of equilibrium points. It is eventually proved that a Hopf bifurcation appears.

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Wing in Ground Effect over a Wavy Surface

Valentin BUTOESCU

Abstract: A vortex method has been used to investigate the effect of a wavy ground on the aerodynamic forces acting on a wing that flies in its proximity. The air is considered inviscid and incompressible. The problem is obviously unsteady, and it is solved both in two and three dimensions. The models used to solve it for 2 and 3D conducted us to systems of integro-differential equations. The solutions were found numerically.

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Design, Manufacturing and Testing of Hybrid Rocket Engines for Aerospace Propulsion

Adrian-Nicolae BUȚURACHE*, Grigore CICAN

Abstract: This paper aims to offer a perspective regarding the design, construction (manufacturing and assembling) and testing of a hybrid rocket engine using gaseous oxygen and paraffin as propellants. NASA CEA software was used in order to obtain the combustion parameters. These parameters were subsequently used to dimension the nozzle of the experimental rocket engine. The imposed thrust was 100 N, after that the rocket engine being optimized based on several parameters. Using the nozzle geometry and the parameters obtained following the combustion reaction between gaseous oxygen and paraffin, numerical simulations were performed using the commercial software CFX-ANSYS, which allowed observing the jet expansion and the flow field. The results obtained from the analytical calculus were validated through the numerical simulations. After defining the nozzle geometry and validating it, the other components of the rocket engine were established in terms of materials and pre-dimensioning. The propellant loads were dimensioned based on the regression rate of the propellant mixture and then manufactured. Finally, several considerations were made regarding the testing and the test bench assembly of the rocket engine.

Key Words: hybrid rocket engine, numerical simulation, regression rate, propellant, gaseous oxygen, paraffin, nozzle

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Thrust and Jet Directional Control Using the Coanda Effect

Alexandru DUMITRACHE*¹, Florin FRUNZULICA^{2,1}, Octavian PREOTU³, Tudor IONESCU^{2,1}

Abstract: The application of the Coanda effect to the directional control of a jet is presented. Deviation of the thrust force by direct flow can be achieved by using the Coanda effect to change the angle of the primary jet engine exhaust nozzle. Since single jet flows or multi-jet flows are extensively applied in conjunction with the Coanda surface, as confined or free jet flows, further insight into complexities involving issues such as the variety of flow structure and the related bifurcation and flow instabilities are provided. The numerical investigations are performed using a RANS solver with an adequate turbulence model and demonstrates a changing of a jet direction. Thus, the conditions and the limits within which one can benefit from the advantages of Coanda-type flows are determined.

Key Words: Coanda effect, Jet vectoring, CFD, Stationary bifurcation

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CFD Based Wind Tunnel Solid Wall Corrections for 2D High Lift Configurations

Mihai Leonida NICULESCU*, Mihai Victor PRICOP,
Marius Gabriel COJOCARU, Maria Cristina FADGYAS

Abstract: The correlation between wind tunnel experimental results and free stream flight results is difficult even for incompressible flows and 2D configurations. This correlation becomes more difficult to be found if the geometrical configuration contains high lift devices such as Kruger and flaps due to the significant amount of separations. Furthermore, the walls of subsonic wind tunnel cause massive wave pressure reflections especially for high blockage, which affect dramatically the aerodynamic coefficients for experimented models. Unfortunately, very few papers and books deal with this delicate subject even if the appearance of subsonic wind tunnel dates over 100 years ago. For this reason, the present paper deals with this complex subject in order to bring some new light in this complex field. This paper is based on an impressive number of CFD simulations for plain airfoils and 2D high lift configurations for wind tunnel and free stream configurations in order to find some useful correlations for INCAS subsonic wind tunnel.

Key Words: Wind Tunnel Correlations, 2D incompressible flow, high lift configurations, CFD.

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Low fidelity models based airfoil optimization

Mihai Victor PRICOP*, Mihai Leonida NICULESCU, Marius Gabriel COJOCARU,
Andra MANTIGHIAN, Ionuț BUNESCU, Adrian DINA

Abstract: Direct airfoil optimization with single or multiple objectives is always required in preliminary aircraft design. Low fidelity flow methods like incompressible potential and incompressible boundary layer are still attractive when used under a genetic algorithm which although relatively slow, enables the identification of the global multi objective solution, naturally handling generic constraints. The work is focused on moderate angles of attack, for flows without large separations, where the current models provide useful certainty. Airfoils to be optimized belong to NACA 4 digit family, with some modifications. Free and constraint optimization methods are used, and it is shown that constraint optimization is fully consistent, bringing solution uniqueness according to the engineering level. Two solvers are used: panel plus flat plate laminar boundary layer solution and a solver from literature. Validation of the solution solver is performed against Xfoil.

Key Words: Airfoil optimization, Genetic algorithm, constraint, panel method.

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Section 2. Equations of Mathematical Physics

Continuum models for crystalline elasto-plastic materials with microstructural defects

Sanda CLEJA-ȚIGOIU*, Victor ȚIGOIU

Abstract: The paper deals with the mathematical description of macroscopic behaviour of materials with defects existing at the microscopic level. We discuss the physical nature and mathematical description of the variables, and we elaborate the constitutive framework. The lattice defects considered here will be dislocations, disclinations and point defects (extra matter or vacuum) including micro voids and micro cracks. We adopt the differential geometry motivation given by Kröner (1992) and de Wit (1981). The dislocations and disclinations, can be modelled by Cartan's torsion of the plastic connection, while the point defects are characterized by the tensorial measure of the non-metricity of the plastic connection. Here the pair: plastic distortion-plastic connection with non-metric property, defines the geometrical properties of the plastically deformed material. The constitutive and evolution equations are coherent and compatible with the free energy imbalance principle, reformulated to be applicable to the elasto-plastic materials with structural defects. The internal dissipated power involves the rates of elastic and plastic distortions, and of measures of defects, as well as the gradients of the corresponding rates, which are power conjugate with the associated macro and micro forces. The free energy is dependent on the elastic and plastic distortions, as well as on the geometrical measure of defects. Boundary value problems have been formulated.

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Probing the Relevance of the Cardiovascular Impedance as a Localized Investigation Technique – A Numerical Experiment

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Abstract: The non-invasive, inexpensive yet accurate medical investigation and diagnosis techniques based on electrical impedance measurements are recently receiving increasing attention. The physical property that bears the information is the electrical conductivity, which differs from tissue to tissue. Depending on the particular positioning of the electrodes, they may provide information on the body composition, *e.g.*, the *Bio-Impedance*, or they may be used for blood flow monitoring, *e.g.*, the *Impedance CardioGraphy* (ICG) and the *Electrical CardioMetry* (ECM) that are sound candidates, recognized as alternatives to the invasive monitoring of hemodynamic parameters. Both methods have been already applied for thoracic impedance measurements, useful in medical diagnosis related to blood circulation through the main artery, the aorta. In this paper we are concerned with the impedance, called *Cardiovascular Impedance* (CVI), which may provide localized information on cardiovascular parameters, as perceived through the arterial blood flow at brachial level. Our approach is the numerical experiment, and we will model the arterial blood flow and its interaction with an externally sourced electromagnetic field using a computational domain constructed out of medical images. The main reason of this study is to probe the relevance of CVI as a monitoring principle for the cardiovascular activity.

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A Predictive Type Approach of Control Delay for Electrohydraulic Servomechanism

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Abstract: A mathematical model of fifth order is considered for the dynamics of electrohydraulic servomechanism. The servomechanism is basically an automatic tracking system. The aim is to design a stabilizing controller taking into consideration a time delay in the control law. This problem is complicated due to some major difficulties of the mathematical model and approached in other works of the authors: critical case for stability, and switching type nonsmooth nonlinearity. Even if we refer only to the associated linear model, the time delay, in this case on the input (control), places the model in the class of infinite-dimensional systems. Our design follows a classical predictor type solution, also used in the so-called Artstein-Kwon-Pierson reduction approach. Thus, by applying the Artstein transformation on the mathematical model, a free-delay system is obtained. Then it is shown how a stabilizing implementable controller for the delay-free reduced model can be designed. This result is to be seen as work in progress, the ultimate goal being the synthesis of a controller for the switching electro-hydraulic servomechanism model.

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On a Class of Nonlinear Evolution Equations for Positive Self-Adjoint Trace-Class Operators

Cecil P. GRÜNFELD

Abstract: We prove the existence and uniqueness of global-in-time solutions to the Cauchy problem for a class of nonlinear von Neumann - Boltzmann equations for the quantum density matrix operator. To this end, we apply recently obtained general results on the initial value problem for an abstract kinetic model in an ordered Banach space (that may not be a lattice).

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On a Boost in Computational Resource Utilization of the Finite Difference Scheme

Ștefan HOTHAZIE

Abstract: This paper describes a generalisation of the finite difference scheme to higher orders of convergence using a reduced stencil. The advantages of using such a numerical scheme are: several orders of magnitude faster computation compared to existing numerical schemes, reduced computer memory utilisation, easily achieving higher orders of convergence and ease of implementation with the aid of computer algebra systems. This paper deals with the rigorous analysis of the numerical scheme and with the techniques of comparing the efficiency of such a numerical scheme to the existing ones. The comparison is done by optimising the highly computationally intensive parts of the code in x86 assembly language, as to eliminate the problem of efficiency of the certain compiler that the user may use. The paper will also include the code and the built executables for further testing and analysis by the reader.

Key Words: numerical scheme, higher order convergence, finite difference, efficiency, faster computation

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Analytical and numerical solution of Riemann Problem for a class of discontinuous hyperbolic systems

Stelian ION, Dorin MARINESCU, Ștefan Gicu CRUCEANU*

Abstract: We investigate the existence of the solution of the Riemann Problem for a system modelling the water flow on a vegetated surface – system of shallow water type equations. In order to define a Riemann solution for such system, we introduce a physical based family of paths that connects the states defining the Riemann Problem. We provide analytical and numerical solutions for certain initial data.

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Non-Newtonian effects in three-layer Hele-Shaw displacements

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Abstract: We study the Saffman-Taylor instability: the displacement of two Stokes liquids in a Hele-Shaw cell is unstable when the displacing fluid is less viscous. In this paper we use an intermediate region IR between the initial liquids, containing a non-Newtonian fluid (a polymer-solute) with a shear-rate depending viscosity. This type of fluid was introduced by Bonn *et al.* [Phys. Rev. Lett. 75, 2132, 1995]. Both shear thinning and shear thickening cases are considered, depending on a specific parameter. The case when the viscosity in IR is dependent on polymer-concentration is also studied. We perform a modal linear stability analysis and get an estimate of the growth constant of perturbations. An improvement of stability is obtained, compared with the Saffman-Taylor case.

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Drop dispensing in a viscous outer liquid

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Abstract: The formation and detachment of Newtonian drops in viscous external liquids is investigated. A global analysis of two necking processes is presented in order to highlight the behaviour of such thinning phenomena, when controlled either by inertia or by viscous effects. Moving detached droplets in an immiscible outer liquid were studied in terms of velocity and drop-travel distance. Theoretical predictions are proposed and compared with experimental data for the volume of the drop and for the subsequent dynamics that follow after detachment. Our investigations point out that the drop rapidly achieves constant velocity, the value of it being in a satisfactory agreement with the model. Both the influence of the flow rate and that of the material properties on drop volume are pursued.

Key Words: capillarity, drop formation, filament thinning, drop detachment

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Section 3. Dynamical Systems

Dynamic model of a hybrid platform, Tandem Quadrorotor X with aerodynamic effects

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Abstract: In this paper, a mathematical model of Tandem Quadrorotor X Hybrid Platform is derived using Newton`s and Euler`s laws. A linearized version of the model is obtained including aerodynamic effects contribution on vertical take-off and landing flight. A parametric study for Vertical Take-Off and Landing mode has been analysed and its stability performances have also been verified with numerical simulation in Matlab/Simulink. The aerodynamic effects have significant contribution in Vertical Take-Off and Landing motion.

Key Words: Hybrid Platform, Quadrorotor, VTOL, aerodynamic effects

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Modelling and Simulation of jet transport motion in terminal phases

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Abstract: In this paper a complex dynamic model representing the motion of aircraft in take-off/landing rolling and transition/approach is developed. Newton – Euler formulation is used to derive the approach and rolling to the moving aircraft and the forces and moments from tire contact with ground are transferred to aircraft body axis. The developed model is structured in the sense that the main system is regarded as an assemble and the characteristics of rolling are modelled by a separated module. A case study for take-off rotation problem has been analysed and the performance of the take-off/landing has been verified on a jet transport aircraft in Matlab/Simulink. A comparison between different estimation of take-off rotating speed has been developed.

Key Words: take-off rotation speed, take-off, landing, transition, approach

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Model of Two-Temperature Convective Transfer in Porous Media

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Abstract: This study is devoted to the asymptotic behaviour of the solution of a convective heat transfer boundary problem in an ε -periodic domain. It consists of two interwoven phases, solid and fluid, separated by an interface on which the heat flux is continuous and proportional with the temperature jump. This first-order jump condition presents a heat transfer coefficient which is assumed ε -periodic, together with the thermal diffusion tensors of the two phases. The fluid flow and its dependence with respect to the temperature are governed by the Boussinesq approximation of the Stokes equations.

We prove the existence and uniqueness properties of the corresponding velocity, pressure and temperature distribution. An L^∞ -estimate of the temperature is obtained. In the case when the Rayleigh number is of unity order, we find by homogenization that the two-scale limits of the solutions verify the most commonly system used to describe local thermal non-equilibrium phenomena in porous media, which, since now, was justified only by volume averaging arguments.

AMS Subject Classifications: 35B27, 76M50, 76Rxx, 74F10, 74Q05

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Effect of the external medium of finite size on the swelling stage of a pulsatory liposome

Dumitru POPESCU

Abstract: A unilamellar lipid vesicle filled with an aqueous solution of an osmotic solute, inserted into a hypotonic medium has a cyclic evolution. For this reason it is called pulsatory liposome. Due to the mechanical tension induced by the osmotic inflow, the lipid vesicle swells up to a critical size, when suddenly only one transient pore appears. Due to this event, the sense of the liposome evolution changes. The liposome relaxes up to the initial size, when the pore closes and another cycle can begins. In this paper we have considered a pulsatory liposome inserted in a closed environment.

The liposome swelling is affected, especially. We have obtained the recurrent series of differential equations which describes the liposome swelling for each successive cycle during the running time of pulsatory liposome. We believe that such liposome may be used, in the future, in medical applications, as a device for controlled drug delivery at ill places, established before. In such ill places, the liposomes will work inside a closed space.

Key Words: Lipid vesicle; Osmotic stress; Transient pore; Drug delivery.

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Asymptotic analysis for a general case of fluid-structure interaction model

Ruxandra STAVRE

Abstract: A two-dimensional time dependent model of an interaction between a thin elastic plate and a viscous fluid described by the non-steady Stokes equations is considered. The problem depends on a small parameter ε that is the ratio of the thicknesses of the plate and of the fluid layer. The Young's modulus of the plate and its density may be great or small parameters equal to some powers (positive or negative) of ε . An asymptotic expansion is constructed and justified for various magnitudes of the rigidity and density of the plate. The limit problems are studied in all these cases. They are Stokes equations with some special boundary conditions modelling the interaction with the plate. It is proved that the difference between the exact solution and a truncated asymptotic expansion with respect to suitable norms is small, that justifies the asymptotic construction.

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Study about ATS surveillance systems in the Flight Information Service

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Abstract: The surveillance environment is in the process of changing from being based on ground radars to using a mix of surveillance technologies, each of which has its appropriate domain. The navigational accuracy and lack of terrestrial constraints offered by a global navigation satellite system based Required Navigation Performance - RNP structure is one in a series of enablers for optimized air route spacing and operational procedures which leverage modern aircraft capabilities and new airport capacities. The expanding application of surveillance separation minima will see an increased capability to deal with air traffic density and complexity in transcontinental, en route airspace. The surveillance systems, as prospective aircraft equipment's that will operate with improved efficiency on current routes and will progressively have more opportunities to operate on flexible flight paths, are analysed in this paper.

Key Words: ATS, Flight Information Service, Operation Procedures, Surveillance Systems

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Integrated Air Traffic Management Systems

Gabriela STROE^{*1}, Irina-Carmen ANDREI²

Abstract: The future Air Traffic Management will be conducted in an environment of significant and continued advances in ground and airborne technologies. The new avionics that will equip the future aircrafts will enhance their flight capabilities and multiple-option forms of efficient and environmentally friendly free-flight, as well as the capability for self-separation and improved cockpit situational awareness in some circumstances. The service models currently in place will need to progressively adapt. The adequate modelling of the efficiency of air transport while using the Air Services Operational Analysis and trajectory - modelling capabilities in purpose to identify the key areas of Air Traffic Management inefficiency and to prioritize and implement the optimized methods, are studied and analysed in this paper. For determining the optimal conditions of flight, it is necessary to check an array that contains real-time air traffic, weather conditions, and aircraft data to uncover post departure opportunities for individual flights to save fuel and improve operational performance.

Key Words: ATM, ATS, airborne technologies, FMS.

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Sliding Mode Control of a Hexacopter

Serena Cristiana VOICU*, Mirela-Madalina BIVOLARU, Adrian Mihail STOICA

Abstract: The paper focuses on the control of a hexacopter, the main purpose being to design a control law able to guarantee robust stability performances with respect to the modeling uncertainties. The hexacopter has several characteristics that give it operational advantages over other types of flying vehicles. The reason of choosing a hexacopter is that in case of external perturbations, such as wind, the hexacopter can be stabilized much easier. Numerical simulations test the stabilization applied on the non-linear dynamic model of the hexacopter. The paper is structured as follows. The first part presents a short introduction about the hexacopter which is an unmanned aerial vehicle with six rotors. The second section of the paper contains preliminaries and problem formulation. After presenting the nonlinear dynamic model of the hexacopter, in the third part the controller design method based on the sliding mode approach is presented. The proposed design methodology is illustrated by a case study in the last section. The paper ends with some concluding remarks.

Key Words: hexacopter, sliding mode control, autonomous UAV, nonlinear control

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Section 4. Mathematical Modeling

Possible Simple Structures of the Actual Universe to Include General Relativity Effects

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Abstract: The general relativity describes the universe properties due to gravity effects by introducing a metric tensor in a Riemann space, in agreement with a mass (or energy-impulse) tensor in order to satisfy the Einstein equation, which also contains the tensor of the space curvature. Applications are done considering that the chosen metric is valid without region limits. In fact, the mass-energy density is variable in universe and therefore the metric should be adapted. Because the real mass-energy distribution is unknown one suggest to start with some possible structures which could represent on average in a first step the actual universe. One conclusion is the decreasing of the gravity effects when one approaches the universe frontier.

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Approach to determine the noise levels inside helicopter cabin. Case study to obtain input data for helicopter cabin noise reduction campaign

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Abstract: The case study presented in this paper is part of the work realized in frame of the PN-III-P2-2.1-BG-2016-0211 programme, project title: „Optimized sound absorbent structures for improved acoustic comfort inside helicopter passenger cabin”, contract 97BG/2016. The project is dedicated to the IAR SA Brasov, which is obliged in order to maintain a good position on the helicopter market, to develop and introduce into the European market noise competitive helicopters. We are aiming to reduce noise inside the cabin, without making changes on its structure or propulsion system, using products based on new concepts of advanced materials that respond to the client's requests. In order to improve the acoustic comfort inside the helicopter cabin, the first step is to obtain the acoustic characterization of the helicopters produced by IAR Brasov SA and after this to create a data base with sound proofing materials created to respond on the needed frequencies. The materials are tested in laboratory in such way to determine the perfect combination to obtain a good correlation between noise characteristics, price, weight and versatile in use.

Practically, the paper will provide the general steps that must be performed, starting to the specific measurements that must be carried out, the materials analyze, the tests performed into laboratory and finalizing with the design of the optimized solution.

Key Words: helicopter cabin, noise levels, noise reduction, sound proofing materials

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Magnetic Nozzles for space plasma thrusters

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Abstract: Plasma propulsion systems are based on the interactions between a compressible ionized gas and magnetic and electric fields applied and induced by the electric charges movement. The particles form, as a whole, a neutral plasma and, under the electrical field's influence, they will move in opposite directions, if a magnetic field is overlapped at the same time, perpendicular to the electric field's direction. In this paper, a study will be conducted regarding the changing parameters characterizing the 1-D flow of an ionized gas in a reaction nozzle, keeping steady magnetic and electric fields along the nozzle. Several plots will be presented, for pressure, density, speed, temperature for various values characterizing the magnetic and electric fields and also for several propellants. The first phase of the study consists of two-step analysis: the first step considers the flow analysis through a channel with constant cross section, equal in length with the measurement unit, for two cases, isothermal flow and adiabatic flow; the second step considers a Laval nozzle for the same conditions specified before. The second phase will consist of numerical simulations made in COMSOL for some of the studies mentioned above and finally, the results will be presented comparatively.

Key Words: Magnetic nozzle, plasma, numerical simulations, electromagnetic field

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Aerodynamics and Chemistry for an Earth Reentry Capsule

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Abstract: The temperature in the front region of a hypersonic vehicle nose can be extremely high, for example, reaching approximately 11 000 K at a Mach number of 36 and altitude of 52 km (Apollo reentry) due to the bow shock wave. Fortunately, the flow is laminar because the density of air at this altitude is very low ($\approx 0.77E-3$ kg/m³). Firstly, this implies that the convective aerodynamic heat transfer (flux) to the vehicle surface is significant smaller than for the turbulent flow according to the Fourier's law. Secondly, the chemistry (dissociation of O₂ and N₂, formation/destruction of NO and ionization) is governed by the Arrhenius equation, which is a formula for the temperature dependence of reaction rates and it is valid only for laminar flows. Thirdly, the thermal and velocity boundary layers are laminar. Unfortunately, very few papers deal with the hypersonic boundary layers over reentry capsules. For this reason, the present paper focusses on hypersonic boundary layer over an Earth reentry capsule.

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Mathematical model for fast drag coefficient estimations with application to small launcher optimisation

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Abstract: In the past years, the small satellite launcher sector has been receiving more and more attention. Developing a conceptual design for a small launcher is a multidisciplinary task, one of the several disciplines that must be analysed being the aerodynamics of the launcher. Because of the complexity of the multidisciplinary optimisation algorithm needed to obtain an optimal small launcher, the aerodynamic assessment of the launcher must be realised very fast. Depending on the complexity of the dynamic models used to simulate the trajectory of the launcher, the number of aerodynamic coefficients varies. For a 3DOF approach, preferred in the conceptual design phase, only the drag coefficient at zero angle of attack is needed. The purpose of this paper is to present a fast mathematical model that can be used to quickly assess the drag coefficient for generic launcher configurations. The tool developed based on this mathematical model can be used separately or integrated in a multidisciplinary optimisation algorithm.

Key Words: aerodynamics, drag coefficient, small launchers, multidisciplinary optimisation, mathematical model

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Incompressible flow past elliptic ring wings of high aspect ratio

Adrian STOICA

Abstract: Prandtl's lifting line model is extended to the elliptic ring wings of high aspect ratio. This model leads to a hypersingular integral equation for which the unknown is the distribution of circulation over the span. The kernel of the equation can be decomposed into a regular part and a singular part. The singular part is the derivative of a Hilbert kernel type. With the aid of a new quadrature formula, for an integral with this type of singular kernel, a numerical solution is obtained. Numerical calculations of the lift and drag coefficients reveal for various eccentricities that the span efficiency factor is over unity in the case of an elliptic ring wing of constant chord length at a constant angle of attack.

Key Words: hypersingular integral equations, Hilbert kernel, quadrature formula, span efficiency factor

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Section 5. Technical Applications

Design Issues in Case of Pumps for Liquid Propelled Rocket Engines

Irina-Carmen ANDREI

Abstract: The intent of this paper is to study significant design issues in case of pumps for liquid propelled rocket engines. The task originates from the need to design a new pump, starting from an already built one, but with given a complete different set of design parameters, with the concomitant compliance to certain restrictions. In concordance with the amplitude of the shaft power and the direction of the inner flow, the types of the pumps can be either axial, centrifugal or diagonal (i.e both axial and centrifugal); also, the pumps can be single-staged or multiple-staged. The focus within this study are the centrifugal single-staged pumps. The potential applications of such pumps are the stages of liquid propelled rocket engines.

Key Words: centrifugal pumps, liquid propelled rocket engines, design

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Numerical investigation of vertical axis wind turbine based on lift-drag forces

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Abstract: The aim of the following research is to provide a comparison between two types of blade airfoils for a vertical axis wind turbine. The first type of blade was generated using a NACA airfoil with the suction side surface removed, while the second one was created using the same airfoil, by removing the pressure side surface this time. The obtained geometries enable the wind turbine to be driven by both lift and drag. During the research, the power coefficient was computed for different TSR values using CFD analysis for turbulent flows (K-omega viscous model), and was used to compare the overall efficiency of the two blade airfoils. The computational domain was generated with ICEM CFD, while for the solver and postprocessing, ANSYS FLUENT was used. For each type of airfoil the aerodynamic coefficients of lift and drag were computed for the optimal value of TSR, further plotting these values to obtain the polar diagram of the blade airfoils.

Key Words: wind turbine, TSR, power coefficient, CFD, blade

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Micro-launch vehicles studies

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Abstract: Development of micro-launch vehicles is more important in the space sector as the market segment of small satellites has been constantly growing in the past years and prospects are for further growing. INCAS is involved in several activities related to micro launch vehicles design in which young researchers take a significant role.

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Flight Control System Design and Analysis of a Light Sport Aircraft with Emphasis on Multibody Dynamics and Aerodynamic Analysis

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Abstract: The present contribution is a companion paper of [9] which presents a preliminary design of the flight control system of a Light Sport Aircraft (based on previous wind tunnel analysis), validation process, aerodynamic simulations in ANSYS CFX and multibody dynamics in Adams. The main purpose of this goal of study is to obtain knowledge on simulation software's, as Ansys and Adams, in order to design, calculate and improve the flight control mechanism of an ultralight aircraft.

Key Words: aircraft design, aerodynamics, wind tunnel, wingtip vortices, multibody-dynamics

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Supersonic and transonic Mach probe for calibration control in the Trisonic Wind Tunnel

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Abstract: A supersonic and high speed transonic Pitot Prandtl is described as it can be implemented in the Trisonic Wind Tunnel for calibration and verification of Mach number precision. A new calculation method for arbitrary precision Mach numbers is proposed and explained. The probe is specially designed for the Trisonic wind tunnel and would greatly simplify obtaining a precise Mach calibration in the critical high transonic and low supersonic regimes, where typically wind tunnels exhibit poor performance. The supersonic Pitot Prandtl combined probe is well known in the aerospace industry, however the proposed probe is a derivative of the standard configuration, combining a stout cone-cylinder probe with a supersonic Pitot static port which allows this configuration to validate the Mach number by three methods: conical flow method – using the pressure ports on a cone generatrix, the Schlieren-optical method of shock wave angle photogrammetry and the Rayleigh supersonic Pitot equation, while having an aerodynamic blockage similar to that of a scaled rocket model commonly used in testing. The proposed probe uses an existing cone-cylinder probe forebody and support, adding only an afterbody with a support for a static port.

Key Words: Supersonic Pitot Prandtl, transonic Mach number measurement, Mach calibration and validation in transonic regimes, calculation method, numeric method

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Demonstrator for Technologies Validation

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Abstract: The aim of the project is to design and build a modular and light multi-engine VTOL vehicle based on three turbojet engines, as test/development platform for the advanced control techniques related to launchers and landers. The vehicle is capable of autonomous flight with scientific/commercial payloads and can also perform endurance flight without payload, for increased flight time. The vehicle is designed to be flexible enough to accept payloads that differ in weight and volume such as measurement hardware used for scientific research that is targeting new and emerging space applications: visual navigation and hybrid navigation. Apart from the standard payload carrying capabilities, the vehicle can also be employed as host/development/qualification vehicle for customer high end avionics systems.

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Fuel level monitoring system by using mechanical guided waves, with application to a mixed flows turbofan engine

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Abstract: The number of the technical applications of the mechanical guided waves has grown substantially in the past decade. One of these applications is the fuel level monitoring. The fuel level is one of the most important parameters that is compulsorily monitored on aircrafts because it is a main safety issue. There are many methods for completing this purpose, but the use of the mechanical guided waves, due to certain advantages, might prove to be promising. This method is using the effect of the fuel amount which is in physical contact with the fuel tank wall, which is subjected to effect of the mechanical guided waves propagation along the wall. A significant advantage of this method resides in the fact that the measurement is proven to be less affected by the fuel tank orientation in space (i.e. during aircraft flight), therefore it can be highly recommended to be used for measuring the fuel level inside the aircraft's fuel tanks. The application of this method to a mixed flows turbofan engine as a propulsion system for a subsonic fighter plane is analyzed in this paper.

Key Words: mechanical guided waves, Lamb waves, mixed flows turbofan engine

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Fuel consumption monitoring based on the tank gas pressure variation method, with application to a mixed flows turbofan engine

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Abstract: The usual fuel level monitoring systems are oriented towards screening the fuel level, by using sensors placed inside the liquid fuel tank. In case that such sensors cannot be placed inside the tank, for different reasons, there are other alternative methods. In this paper is investigated a method which is using sensors placed outside the fuel tank and consists in monitoring the variations of the gas pressure from the tank, being based on the modifications in gas volume consequent to the changes in liquid fuel volume. This method could be either passive (when the gas pressure is monitored, based on the determined variations in air volume) or active (when it is produced a controlled variation in gas volume while gas pressure is monitored). For both cases (i.e. active or passive method), there is no requirement for placing the sensors inside the fuel tank; instead, the location of the sensors could be anywhere along the connected pipes to the fuel tank. An important advantage of this method consists in the fact that the results of the measurements are not influenced by the orientation of the fuel tank with respect to the direction of the gravity force (in case of fighter or combat aircrafts), neither by the absence of gravity (in case of other space vehicles). The application of this method to a mixed flows turbofan engine as a propulsion system for a subsonic fighter plane is investigated in this paper.

Key Words: fuel consumption monitoring, tank gas pressure variation, mixed flows turbofan engine

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Extended Abstracts

Integral Method for Static, Dynamic, Stability and Aeroelastic Analysis of Beam like Structure Configurations

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Abstract: This work presents a synthesis of the use of an integral approximate method based on structural influence functions (Green’s functions) concerning the behavior of beam like structures. The domains of the use of this integral method are static, dynamic, stability and aeroelasticity. The integral method starts from the differential equations governing the bending or/and torsional behavior of a beam. These equations are put in integral form by using appropriate Green’s functions, according to the boundary conditions. Choosing a number of n collocation points on the beam axis, each integral are then computed by a summation using weighting numbers. This approach is suitable for conventional Euler-Bernoulli beams and also for the thin-walled open or closed cross-section beams which can have bending-torsion coupling. Generally, for a static analysis this approach leads to a linear system of equations (the case of the lift aeroelastic distribution analysis) or to an eigenvalues and eigenvectors problem in the case of dynamic, stability or divergence analysis.

Key Words: Integral Method, Green Functions, Collocation, Static, Dynamic, Stability, Aeroelasticity.

1. INTRODUCTION

The use of the structural influence functions (Green’s functions) in the structural and aeroelastic analysis are presented for example in [1].

In Romania this type of approaches are widely used for the fixed wing aeroelastic problems in the works of professor A. Petre [2, 3].

In the case of the rotating beams and blades the method using Green’s functions was presented for simple configurations in [4, 5].

The coupled bending vibration analysis and then the coupled bending-bending-torsion vibration analysis in the case of pretwisted blades were presented in [6, 7].

The papers [8, 9] concern the dynamic analysis of rotating beams with tip mass. The method of Green’s functions was then applied for composite beams [10, 11].

Aspects concerning the aeroelastic analysis of wings are presented in the work [12]. New developments of the methods based on Green’s functions in dynamic analysis of beams and blades are recently reported in [13, 14].

In the case of the dynamic, stability or aeroelastic analysis this method leads to an eigenvalues and eigenvectors problem.

This paper presents the general formulation of the method and two simple examples, discussed in comparison with analytical results.

2. STATIC ANALYSIS

The bending behavior of a straight beam, having the length L and loaded transversally by the distributed force $p(x)$, can be described by a differential equation:

$$[EI(x)w''']' = p(x) \quad (1)$$

It can take the integral form, [2]:

$$w(x) = \int_0^L G_w(x, \xi) p(\xi) d\xi \quad (2)$$

The previous equation is based on the Green’s function $G_w(x, \xi)$ representing the bending deflection $w(x, \xi)$ at distance x due to a unit force applied at ξ (Fig. 1).

The differential equation governing the Saint Venant torsional behavior of a beam, having the length L and loaded by the distributed torsion moment $m_t(x)$, is:

$$[GJ(x)\phi'] + m_t(x) = 0 \tag{3}$$

It can be written in the integral form:

$$\phi(x) = \int_0^L G_t(x, \xi) m_t(\xi) d\xi \tag{4}$$

using the Green's function $G_t(x, \xi)$ representing the twist deflection angle $\phi(x, \xi)$ at distance x due to a unit torsion moment applied at location ξ (Fig.1).

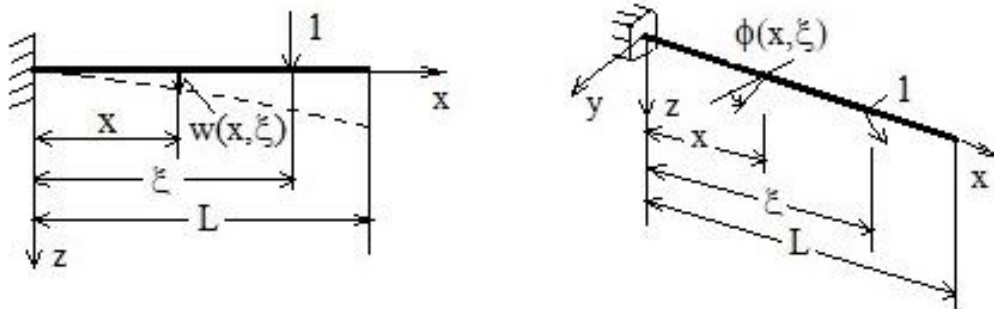


Fig. 1 – Physical significance of Green's functions

The material of the beam is considered metallic, isotropic having the longitudinal elastic modulus E and the shear modulus G .

The notations $I(x)$ and $J(x)$ are for the moment of inertia of the cross section of the beam and respectively the torsional stiffness constant.

The integrals involved in such type of approach can be approximated by a summation using n collocation points ξ_i with $f_i = f(\xi_i)$:

$$\int_0^L f(\xi) d\xi = \sum_{i=1}^n f_i \cdot W_i \tag{5}$$

where W_i are weighting numbers corresponding to Simpson's method of integration adopted here.

The equations (2) and (4) give the possibility to obtain the static bending and torsion deflections for known distributed force $p(x)$ and distributed torsion moment $m_t(x)$.

Aerodynamic loads are distributed loads, other effects like concentrated forces, concentrated moments or discrete attachments can be also introduced.

3. DYNAMIC ANALYSIS

For example the differential equation in the case of the transverse free vibrations of a rotating beam has the following form:

$$[EI(x)w'']' = m(x)\omega^2 w + [T(x)w'] \tag{6}$$

where the axial force (tension due to the angular velocity of rotation Ω) is:

$$T(x) = \Omega^2 \int_x^L m(\xi_1) \xi_1 d\xi_1 \tag{7}$$

The term containing $T(x)$ gives the stiffening effect due to the rotation. In the equation (6), $m(x)$ is the mass of unit length of the beam and ω is its natural circular frequency.

This equation can be considered of the form (1). It takes a matrix form using n collocation points ξ_i and the relations (2), (5):

$$\{w\} = \omega^2 [G_w] [W] [M_1] \{w\} + \Omega^2 [G_w] [W] [M_{in}] [D_2] \{w\} - \Omega^2 [G_w] [W] [M_x] [D_1] \{w\} \tag{8}$$

In the previous equation:

$[G_w]$ is a matrix containing the measured or calculated influence coefficients $G_w(\xi_i, \xi_j)$,

$[W]$ is a weighting matrix depending on the integration method (Simpson, here),

$[D_1]$, $[D_2]$ are differentiating matrices based on central difference operator,

$[M_1]$, $[M_{in}]$ and $[M_x]$ are diagonal matrices with the values $m(x)$, $\int_x^L m(\xi)\xi d\xi$ and $m(x)\cdot x$ respectively along the main diagonal.

The equation takes the form:

$$\{w\} = \omega^2 [G_1] \{w\} + \Omega^2 [[G_2] - [G_3]] \{w\} \quad (9)$$

which is a standard eigenvalue problem:

$$[[A] - \omega^2 [I]] \{w\} = \{0\} \quad (10)$$

$$[A] = [G_1]^{-1} [[I] - \Omega^2 [G_2] + \Omega^2 [G_3]] \quad (11)$$

a matrix of $n \times n$ dimension and $[I]$ an unity matrix having also the dimension $n \times n$.

The dimension of the eigenvalue problem can be reduced by the use of collocation functions corresponding to the boundary conditions.

The results are the natural circular frequencies $\omega_i = 2\pi f_i$, with f_i the natural frequencies [Hz]. An example is also presented in comparison with analytical results for a clamped-free non-rotating uniform beam.

4. STATIC AEROELASTICITY ANALYSIS

In the work [16] are presented examples of the use of this integral formulation in the wing divergence analysis. The wing is considered like a straight clamped-free beam. In the torsion divergence analysis one can also obtain a standard eigenvalue problem:

$$[[A_2] - \lambda(q)[I]] \{\phi\} = \{0\} \quad (12)$$

The eigenvalues λ depend on the dynamic pressure $q = \rho v^2/2$ with ρ and v the air density and air velocity respectively. The first minimum eigenvalue λ_1 gives the divergence velocity v_D .

Similar formulation can be obtained in the case of the wing bending divergence analysis.

5. STABILITY ANALYSIS

One consider here the standard problem of the buckling of a pin-ended straight beam subjected to an axial compression force P . The equation governing the bending displacements is:

$$[EI(x)w''']' = -Pw'' \quad (13)$$

Using the same steps, the matrix form of the last equations is:

$$\{w\} = -P[G_w][W][D_2] \{w\} = -P[G] \{w\} \quad (14)$$

in the case of the use of collocation points.

This represents an eigenvalue problem:

$$[[A_1] + P[I]] \{w\} = \{0\} \quad (15)$$

where $[A_1] = \text{inv}[G]$.

The eigenvalues of the matrix $[A_1]$ give the critical buckling loads ($\lambda = -P_c$). An example is then presented in comparison with analytical results, for a pin-ended straight uniform beam, using collocation points and also the collocation functions approach.

6. CONCLUSIONS

The present work presents the static, dynamic and stability analysis of several simple beam configurations using an integral formulation (I.F.) based on the use of *structural influence functions* (Green's functions).

These functions are computed by specific methods of *Strength of materials* using the fact that they can be interpreted as displacement in a point of a beam due to an unit force applied in another point.

The presented integral approach needs the use of a number of *collocation points*. For the numerical integration, *integration matrices* based of Simpson's method of integration are here employed. *Differentiating matrices* are also necessary in the case of rotating beams and buckling analysis. In these cases, the use of *collocation functions* can leads to a better accuracy of the calculations.

The two simple examples reported here show a good agreement with the analytical results. This formulation can be also used in the case of non-uniform beams or for other boundary conditions, using appropriate Green functions determined numerically or by experiment.

The accuracy depends on the number of the used collocation points and on the number and precision of collocation functions.

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Thrust and Jet Directional Control Using the Coandă Effect

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Abstract: The application of the Coandă effect to the directional control of a jet is presented. Deviation of the thrust force by direct flow can be achieved by using the Coandă effect to change the angle of the primary jet engine exhaust nozzle. The numerical investigations are performed using a RANS solver with an adequate turbulence model and demonstrates a changing of a jet direction. Thus, the conditions and the limits within which one can benefit from the advantages of Coandă-type flows are determined.

Key Words: Coandă effect, Jet vectoring, CFD

INTRODUCTION

The Coandă effect were original observed for an external flow but the same phenomenon occurs in channel flow. This effect consists of the tendency of a jet to remain attached to a sufficiently long/large convex surface (figure 1).

Flows deflected by a curved surface have caused great interest in the last fifty years. Major interest in the study of this phenomenon is caused by the possibility of using this effect for aircrafts with short take-off and landing, for thrust vectoring. It is also used in applications involving mixing two or more fluids, noise attenuation, ventilation, etc.

After leaving the channel, the jet entrains through the friction effects the particles of the environment, in the zone where there is no flap and the fluid particles between the jet and the flap.

If the flap is long enough, the place of the entrained particles in the domain between the flap and the jet can no longer be taken by the outer particles and the low pressure thus creates a flow deflection in the direction of the flap.

In other words, if the surface is curvilinear convex, then a combination of skin friction, viscosity, and 'void effect' (near surface) results in a centripetal acceleration of the affected fluid, coexistent with a reduction in pressure because of inertia effects.

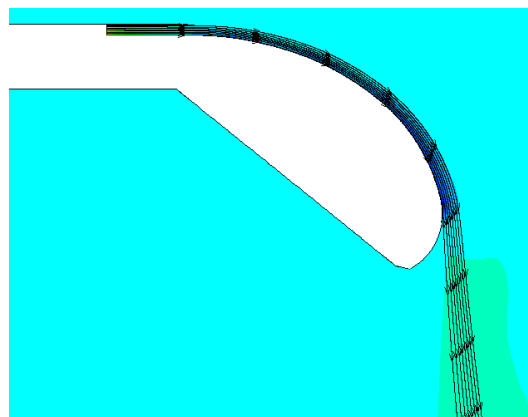


Fig. 1 The Coandă effect

In the paper we present some applications of the Coandă effect in engineering, such as the jet vectoring. Investigations were made using Computational Fluid Dynamics, and the results highlight the positive and negative aspects of analyzed technical solutions.

THRUST AND JET VECTORING

Mechanical deflection of the thrust involves engine nozzle deflection and, thus, physically modify of the main flow direction [2].

Fluid deflection of the thrust involves fluid injecting or removal from the boundary layer of a main jet to allow deflection.

Although a mechanical guidance system of the thrust is effective, it can be complex, difficult to integrate and aerodynamically inefficient [3].

A deflection system of thrust fluid has the advantage of being easy, simple, inexpensive and without moving parts (fixed geometry), and can be implemented, taking advantage of a minimum radar detection of aircraft.

A guidance fluidic system of the thrust must, however, can be implemented in the initial phase of the design process, unlike a mechanical system that can be adapted for aircraft already in operation.

The ones discussed in the literature are different types of techniques targeting fluid orientation of the thrust by the control using shock wave direction, by the direct or reverse flow control on Coanda surfaces (figure 2).

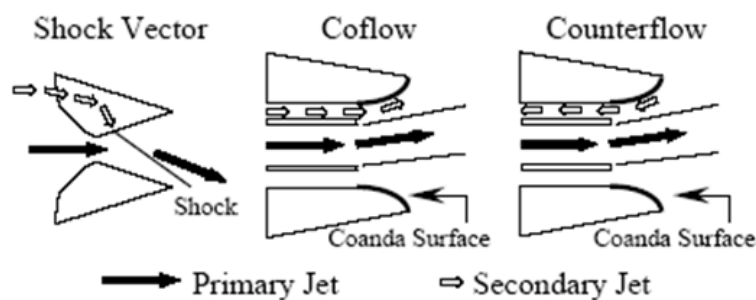


Fig. 2 Types of thrust's fluid deflection [4]

Deviation of the thrust force by direct flow can be achieved by using Coanda effect to change the angle of the primary jet engine exhaust nozzle.

Due to Coandă surface, secondary jet stops triggered on the near surface (figure 3). The resultant force of the thrust vector, $F_{z,lv}$ produces a pitch moment, $M_{z,lv}$ around the airplane center of gravity, allowing the plane to be stabilized during the flight.

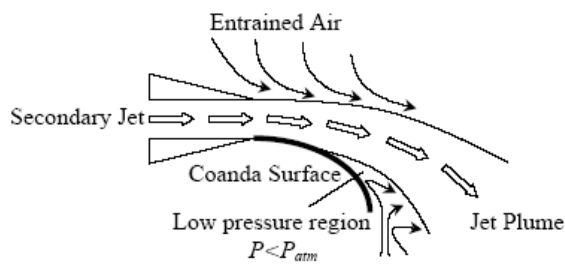


Fig. 3 Coanda effect (the incident jet tends to remain attached to the convex curved surface)

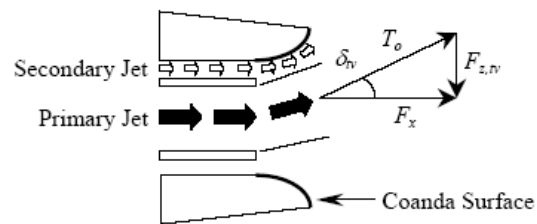


Fig. 4 The concept of thrust deviation - inverse flow method

This force is reported to T_0 to obtain the thrust coefficient

$$C_z = \frac{F_{z,lv}}{T_0} \quad (1)$$

where C_z depends on the angle of the thrust vector according to the equation

$$\delta_{lv} = \tan^{-1} C_z \quad (2)$$

One must notice that the deviation of the thrust force implies a change of the drag force. When $d_{lv} > 0$, the real force of the primary jet will be smaller than the resultant thrust force $F_x < T_0$.

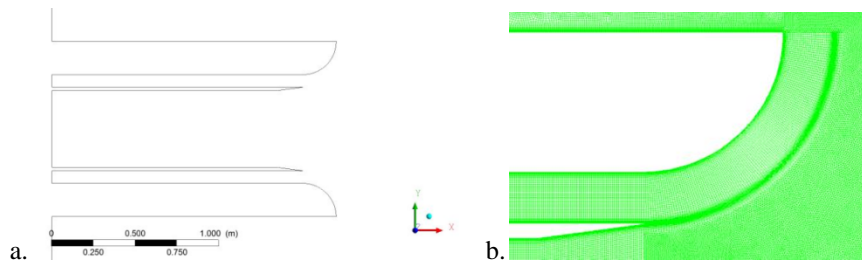
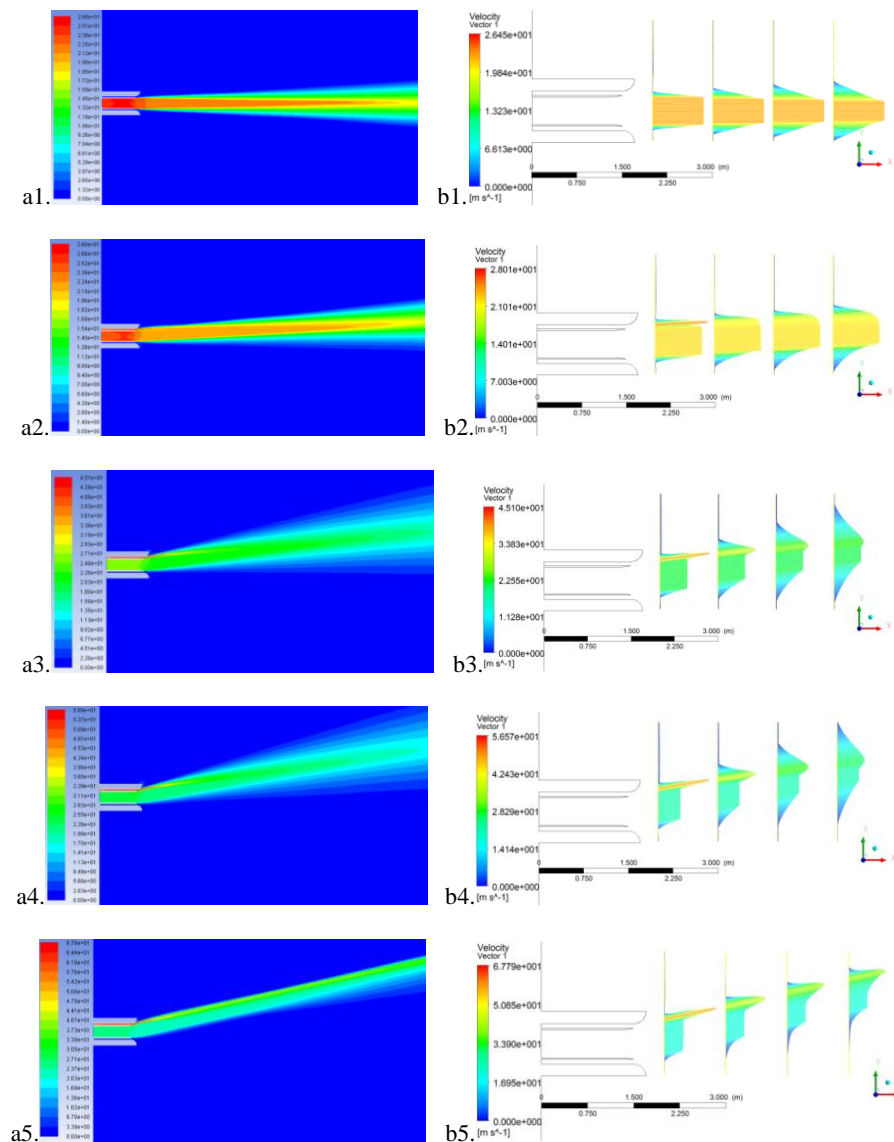


Fig. 5 The model used to highlight the change of the jet direction: *a.* the model geometry; *b.* detail of the computational grid

The effect of the secondary flow on the main jet in the presence of a convex surface can be evidenced by numerical flow simulation on the configuration shown in figure 5. The RANS model with the second order spatial approximation model for flow equations, supplemented with the $k-\omega$ SST turbulence model, is used. The working fluid was the air, and the flow regime is assumed to be incompressible. The main jet has a speed of 25 m/s, with a moderate turbulence degree. By varying the secondary jet velocity (considered only the upper jet in the presented geometric configuration), the results shown in figure 6 are obtained.

Figures 6.a1-b1 correspond to a speed $V_s = 0$ m/s for the secondary jet, 6.a2-b2 for $V_s = 25$ m/s, 6.a3-b3 for $V_s = 40$ m/s, 6.a4-b4 for $V_s = 50$ m/s, 6.a5-b5 for $V_s = 60$ m/s and 6.a6-b6 for $V_s = 70$ m/s. From figure 6 it is observed that with the increase of the secondary jet velocity the deflection of the jet increases and the shape of the velocity profile in the main jet changes from a "full" velocity profile specific to the free jet to an asymmetric velocity profile with the maximum moved towards "the secondary jet". The deflection of the jet is "sustained" also by the shape of the exhaust surface (quarter circle).



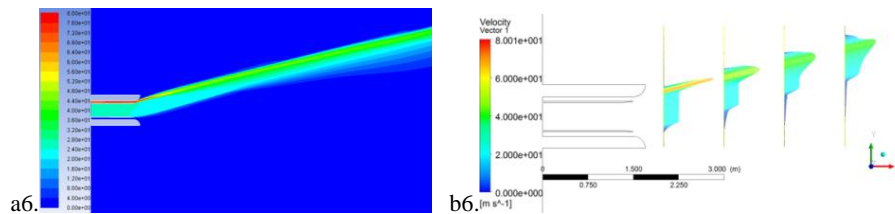


Fig. 6 The effect of secondary jet on the primary jet ($V_j=25$ m/s), the flow velocity field (a1-a6) and the velocity profiles (b1-b6) for 4 positions (2 m, 3 m, 4 m si 5 m)

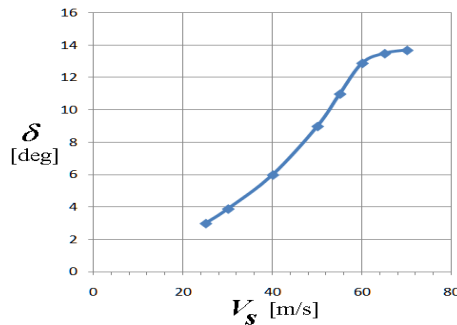


Fig. 7 The deflection of the main jet as function of the velocity of the secondary jet

Figure 7 shows the variation of the main jet angle deflection in relation to the secondary jet velocity. Secondary jet is accelerated over a Coandă surface, causing the appearance of local pressure drop and the pressure gradient “perpendicular” to the main jet axis. This effect correlated with the friction effects leads to an increase in the fluid flow from the main jet to the secondary jet leading to the change of the direction of the jet. Increasing the secondary jet velocity and the radius of the curvature of the Coandă surface, lead to a greater deflection of the direction of the main jet and obtaining the effect of the thrust vectorization produced by the jet. It is noted that for a secondary jet speed of over 60 m/s, the entrainment effect of the secondary jet decreases, the main jet being deflected very little or not at all.

CONCLUSIONS

In this paper we have presented several applications of the Coandă effect in civil engineering/aeronautics regarding the thrust or jet vectoring. We have numerically analyzed the effect of secondary jet on the primary jet and the deflection of the main jet as function of the velocity of the secondary jet has been obtained. Thus, the conditions in which the Coandă effect can successful be used to produce a controlled deflection of a traction have been determined. The effect of a secondary jet with a speed of over 60 m/s has a little effect on the main jet deflection.

A possible further investigation can be done using synthetic jets to obtain an augmenting vectoring effect. The Coandă effect offers extensive perspectives in aeronautics and non-aeronautics applications.

ACKNOWLEDGMENTS

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An Approach of Control Delay for Electrohydraulic Servomechanism

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Abstract: A mathematical model of fifth order is considered for the dynamics of electrohydraulic servomechanism. The problem proposed to solve in this paper is that of delay in control. This problem is complicated due to some major difficulties of the mathematical model which were also studied in other works of the authors: critical case for stability, and switching type nonsmooth nonlinearity. Even if we refer only to the associated linear model, the time delay, in this case on the input (control), places the model in the class of infinite-dimensional systems. To counteract the delay on control, a predictor feedback is used.

Keyn Words: delay on control, electrohydraulic servomechanism, stability, feedback predictor, Artstein reduction method

I. INTRODUCTION

Electrohydraulic servomechanisms (EHS) are used to command the primary flight controls of the airplane. These are automatic tracking systems of some input variables, in the present case, electric signals from the automatic pilot. EHS provide a great force amplification at output, also ensuring the fast and accurate movement of these control surfaces. It is well known that delayed systems have a lot of attention lately, given the many applications in engineering practice.

II. MATHEMATICAL MODEL

The following mathematical model of EHS, slightly modified from the usually model considered in [1], [2], [3], (see Fig. 1), is taken into account

$$\begin{aligned} \dot{x}_1 = x_2, \dot{x}_2 &= \frac{(-kx_1 - fx_2 + Sx_3 - Sx_4)}{m}, \dot{x}_5 = -\frac{1}{\tau_{SV}}x_5 + \frac{k_{SV}}{\tau_{SV}}u(x(t-h)), C := c_d w \sqrt{\frac{2}{\rho}}, y_0 = x_1 \\ \dot{x}_3 &= \frac{B}{V_0 + Sx_1} \left(C|x_5|\operatorname{sgn}[p_s(1 + \operatorname{sgn}x_5) - 2x_3] \sqrt{\frac{|p_s(1 + \operatorname{sgn}x_5) - 2x_3|}{2}} - Sx_2 - k_{il}(x_3 - x_4) \right) \\ \dot{x}_4 &= \frac{B}{V_0 - Sx_1} \left(C|x_5|\operatorname{sgn}[p_s(1 - \operatorname{sgn}x_5) - 2x_4] \sqrt{\frac{|p_s(1 - \operatorname{sgn}x_5) - 2x_4|}{2}} + Sx_2 + k_{il}(x_3 - x_4) \right) \end{aligned} \quad (1)$$

The notations refer to: input u (the control variable) and output y_0 . The state variables are: x_1 , the load displacement; x_2 , the load velocity; x_3, x_4 the pressures in the hydraulic cylinder chambers; x_5 , the EHSV spool valve displacement (opening). The involved constants are: p_s , the supply pressure; m , the equivalent inertial load of primary control surface reduced to the actuator rod; f , the combined coefficient of the damping and viscous friction forces on the load and the cylinder rod; k , an equivalent aerodynamic elastic force coefficient; S , the effective area of the piston; k_{il} the coefficient of internal leakages between hydraulic cylinder chambers; V_0 , the cylinder semivolume; B , the bulk modulus of hydraulic oil; τ_{SV} , the servovalve time constant; k_{SV} proportionality coefficient relating the input voltage to servovalve to valve displacement; c_d , the discharge coefficient; w , the valve port's width; ρ , the hydraulic oil density.

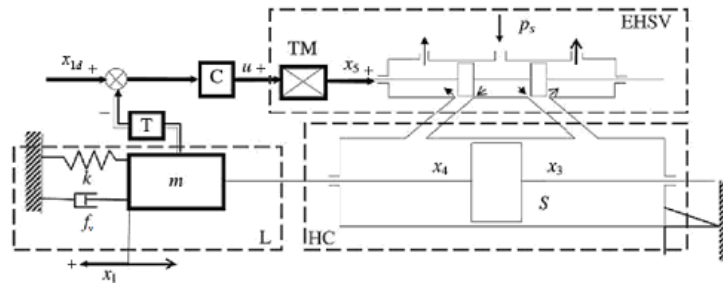


Fig. 1 – Block diagram of the servovalve controlled EHS. HC: hydraulic cylinder with piston; L: load; C: controller; T: transducer; TM: torque motor; EHSV: electrohydraulic servovalve

EHS is basically a tracking system of references such as $x_{1d}(t)$ (Fig. 1). The statement of the tracking problem is the following [2]: *Suppose the system's regulated output $y_0 := x_1(t)$ is required to track some desired output, the reference signal $y_d := x_{1d}(t)$. Provide a state feedback control law $u(t) := u(\mathbf{x}(t))$ that guarantees the asymptotic convergence $\lim_{t \rightarrow \infty} (x_{1d}(t) - x_1(t)) = 0$.*

The *stabilization problem*, in which the desired outputs are the equilibria points of the system, is defined as a special case of the tracking problem: *Suppose that the initial condition of the system's state vector is different from a certain equilibrium point. Provide a state feedback control law $u(t) := u(\mathbf{x}(t))$ that brings the system solution to that equilibrium point. More specifically, the regulated output $x_1(t)$ is required to approach the desired constant value x_{1d} , the first component of some equilibrium point $\lim_{t \rightarrow \infty} (x_{1d} - x_1(t)) = 0$.*

This article focuses on a problem of equilibrium point stabilization.

III LINEAR MATHEMATICAL MODEL

It is shown that by introducing a coefficient of leakage k_{il} between the hydrocylinder chambers in (1), the matrix of the linearized system is Hurwitz. In the paper [3], a critical stability problem, derived from a mathematical model without internal leakage, was treated in principle. It is known that the synthesis of an LQR law, for example, for the system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}_c u$ is conditioned by the controllability, or at least by the stabilizability, of the matrix pair $(\mathbf{A}, \mathbf{B}_c)$.

Control matrix \mathbf{B}_c is in our case a column vector with the first four elements zero and the fifth element equal with k_{SV}/τ_{SV} . Even in form (1), the mathematical model of EHS remains a) strongly nonlinear, b) with switching due to directional change of servovalve ports opening, which is described by $\text{sgn } x_5$, c) and with delayed on control.

Let's choose the case $x_5 > 0$ and let's calculate the poles configuration of matrix \mathbf{A} . The system (1) is written in the form $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, u)$, respectively

$$\begin{aligned} \dot{x}_1 &= x_2; \quad \dot{x}_2 = \frac{-kx_1 - fx_2 + Sx_3 - Sx_4}{m}, \quad \dot{x}_3 = \frac{B}{V_0 + x_1} (Cx_5\sqrt{p_a - x_3} - Sx_2 - k_{il}(x_3 - x_4)) \\ \dot{x}_4 &= -\frac{B}{V_0 - x_1} (Cx_5\sqrt{x_4} - Sx_2 + k_{il}(x_3 - x_4)), \quad \dot{x}_5 = -\frac{1}{\tau_{SV}}x_5 + \frac{k_{SV}}{\tau_{SV}}u_1(x(t-h)) \end{aligned} \quad (2)$$

The equilibrium point (x_e, u_e) is defined by the solution of the system $f(x_e, u_e) = 0$. The solution is as follows: $\hat{x}_1 = x_{01} \neq 0$, $\hat{x}_2 = 0$, $\hat{x}_3 = \frac{kx_{01}}{S} + p_a p$, $\hat{x}_4 = p_a p$, $\hat{x}_5 = x_{05}$, $\hat{u}_1 \neq 0$, with $|x_{01}| < \frac{V_0}{S}$, $p \in (0, 1)$, $p_a(1-p) - \frac{kx_{01}}{S} > 0$, $p_a p + \frac{kx_{01}}{S} > 0$, \hat{x}_5 is given by $C\hat{x}_5\sqrt{p_s - \hat{x}_3} - k_{il}(\hat{x}_3 - \hat{x}_4) = 0$, $-C\hat{x}_5\sqrt{\hat{x}_4} + k_{il}(\hat{x}_3 - \hat{x}_4) = 0$ for $x_5 > 0$. Matrix \mathbf{A} calculated at the equilibrium point is represented in (3).

To illustrate the properties of the pair $(\mathbf{A}, \mathbf{B}_c)$, consider the following design data, representing an EHS integrated in the aileron control chain of the Romanian jet fighter IAR99 [2], [3], [4]: $m = 0.03 \text{ daN} \times \text{s}^2/\text{cm}$, $f = 3 \text{ daN} \times \text{s}/\text{cm}$, $k = 300 \text{ daN}/\text{cm}$, $S = 10 \text{ cm}^2$, $c_d = 0.63$, $V_0 = 30 \text{ cm}^3$, $p_a = 210 \text{ daN}/\text{cm}^2$, $B = 13000 \text{ daN}/\text{cm}^2$, $\rho = 8.5 \times 10^{-7} \text{ daN} \times \text{s}^2/\text{cm}^4$, $k_{SV} = 2 \cdot 10^{-2} \text{ cm}/\text{V}$ (meaning an equivalent valve port width $w = 0.85 \text{ mm}$ and a maximal opening length of rectangular valve port $x_{5\max} = 1 \text{ mm}$ at maximal valve input voltage $u_{\max} = 10 \text{ V}$), $k_{il} = 0.04 \text{ m}^5/(\text{Ns})$ and $\tau_{SV} = 1/573 \text{ s}$.

The equilibrium point is given by $\hat{x}_1 = 0.5 \text{ cm}$, $\hat{x}_2 = 0 \text{ cm}$, $\hat{x}_3 = 112.5 \text{ daN}/\text{cm}^2$, $\hat{x}_4 = 97.5 \text{ daN}/\text{cm}^2$, $\hat{x}_5 = 7.4 \times 10^{-4} \text{ cm}$, $\hat{u} = 0.0370 \text{ V}$ and $p = 0.4643$.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{k}{m} & -\frac{f}{m} & \frac{S}{m} & -\frac{S}{m} & 0 \\ 0 & -\frac{BS}{V_0 + S\hat{x}_1} & \frac{BC\hat{x}_5}{2\sqrt{\hat{x}_3}(V_0 + S\hat{x}_1)} - \frac{Bk_{il}}{V_0 + S\hat{x}_1} & \frac{Bk_{il}}{V_0 + S\hat{x}_1} & \frac{BC\sqrt{\hat{x}_3}}{V_0 + S\hat{x}_1} \\ 0 & \frac{BS}{V_0 - S\hat{x}_1} & \frac{Bk_{il}}{V_0 - S\hat{x}_1} & \frac{-BC\hat{x}_5}{2\sqrt{p_a - \hat{x}_4}(V_0 - S\hat{x}_1)} - \frac{Bk_{il}}{V_0 - S\hat{x}_1} & \frac{BC\sqrt{p_a - \hat{x}_4}}{V_0 - S\hat{x}_1} \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_{SV}} \end{pmatrix} \quad (3)$$

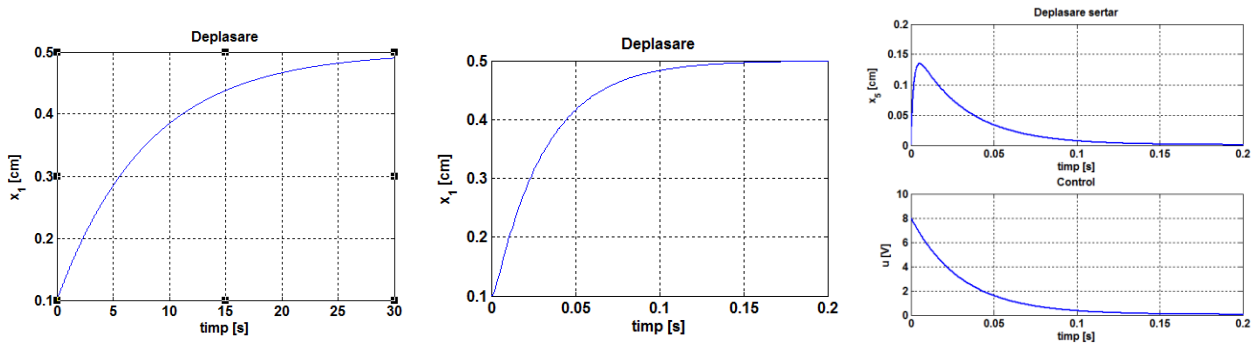


Fig. 2 – Left: Linear system without control; middle: with LQR control; right: the nonlinear associated model

Thus, the matrix \mathbf{A} has the next set of eigenvalues, for $k_{il} = 0$: $0, -50 \pm 1726i, 0, -573$. The matrix pair $(\mathbf{A}, \mathbf{B}_c)$ does not meet the controllability/stabilizability conditions, so an LQR control law cannot be achieved [4], [5]. Taking $k_{il} = 0.04 \text{ m}^5/(\text{Ns})$, it is found that the matrix pair $(\mathbf{A}, \mathbf{B}_c)$ is controllable, so it is stabilizable, too, and the eigenvalues in open loop are $\lambda = (-136.3 \pm 1726.1i, -0.6, -130, -573)$. The closed loop eigenvalues are $\lambda = (-136.3 \pm 1726.2i, -32.3, -130, -572.3)$. It was considered $R = 0.0025$, \mathbf{Q} is the zero matrix excepting $Q(1,1) = 1$. It was obtained the following gain $\mathbf{K} = (19.5649; 0.0009; 0.0026; -0.0019; 2.7111)$. The equilibrium is disturbed by changing $x_{01} = 0.5 \text{ cm}$ in $x_{02} = 0.1 \text{ cm}$. A representative simulation is shown in Fig. 2.

IV. COUNTERACTING THE DELAY BY THE METHOD OF PREDICTED FEEDBACK CONTROL

Although basically we are dealing with a switching system (1), for limited space considerations, we will refer to a generalized linearized system with delay on control and with equilibria translated into zero

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_c u(t-h), u(t) = u_0(t), \forall t \in [-h, 0), \mathbf{x}(0) = \tilde{\mathbf{x}}_0 \quad (4)$$

In a recent paper [6], to counteract delays on control it is proposed a so-called feedback predictor [7], which is clearly a solution of the system $\dot{\mathbf{x}}(t+h) = \mathbf{A}\mathbf{x}(t+h) + \mathbf{B}_c u(t)$

$$x_p(t) := \mathbf{x}(t+h) = e^{\mathbf{A}h} \mathbf{x}(t) + \int_{t-h}^t e^{\mathbf{A}(t-s)} \mathbf{B}_c u(s) ds = e^{\mathbf{A}h} \mathbf{x}(t) + \int_{-h}^0 e^{-\mathbf{A}s} \mathbf{B}_c u(s+t) ds \quad (5)$$

The idea is to apply in (4) the following predicted feedback control deduced from (5), with feedback gain \mathbf{K}

$$u(t) = \mathbf{K}\mathbf{x}(t+h) = \mathbf{K} \left(e^{\mathbf{A}h} \mathbf{x}(t) + \int_{-h}^0 e^{-\mathbf{A}s} \mathbf{B}_c u(s+t) ds \right) \quad (6)$$

This procedure is connected with the so-called method of reduction [8] and leads to the closed loop system

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_c u(t-h) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_c \mathbf{K} e^{\mathbf{A}h} \mathbf{x}(t-h) + \mathbf{B}_c \mathbf{K} \int_{-h}^0 e^{-\mathbf{A}s} \mathbf{B}_c u(t+s-h) ds \quad (7)$$

which contains a homogeneous part and a non-homogeneous part. The stability of the system (4) is determined by the roots of characteristic equation $\det(sI - A - A_1 e^{-hs}) = 0$ associated to the homogeneous part $\dot{x}(t) = Ax(t) + A_1 x(t-h), x(t) = \varphi(t), -h \leq t < 0, A_1 := B_c K e^{Ah}, (*)$.

Proposition [9], [10]. *Based on the direct method of Liapunov, the stability of the system (*), and hence of the system (7), is decided by the existence of at least one solution T of the nonlinear algebraic matrix equation*

$$e^{(A+T)h} T = A_1, A_1 := B_c K e^{Ah} \quad (8)$$

After a rearranging of the terms, by taking into account the commutativity of A and e^A matrices, the equation (8) is so rewritten $hT e^{hT} = D, D := h e^{-hA} A_1$. A solution T based on matrix Lambert Function W [11] is searched numerically using a MATLAB algorithm recently proposed in the paper [12]. Taking a reasonable $h = 10^{-3}$ s, the following solution is obtained

$$T = \begin{bmatrix} -9.38 - 0.00005 - 0.0012 & 0.0009 - 1.31; & 27519.63 & 1.43 & 3.55 - 2.65 & 3833.81; & -58467.4 \\ -3.05 - 7.55 & 5.62 - 8145.21; & 84025.45 & 4.38 & 10.85 - 8.08 & 11705.75; & 385.67 & 0.02 & 0.05 \\ -0.037 & 53.72 \end{bmatrix}$$

V. CONCLUSIONS

It is interesting to note that a control delay problem is solved by transferring the delay on the state and by analysing a corresponding problem of stability of a linear system with state delay. For reasons of space limitation, the question of stability of the non-linear switched system was left for an another paper.

ACKNOWLEDGMENT

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Probing the Cardiovascular Impedance as a Localized Investigation Technique. A Numerical Experiment

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Abstract: This paper aims to evaluate cardiovascular impedance (CVI) as a clinically potential mean for the noninvasive assessment of several hemodynamic parameters. We introduce a mathematical model for CVI technique aimed to elicit electrical bio-impedance (EBI) cardio-vascular indices at the brachial (arm) level, its numerical implementation, and numerical simulation results.

Key Words: Cardio-vascular impedance, Brachial arterial flow, Numerical Modeling.

1. INTRODUCTION

Cardiovascular diseases are sought of as the main cause of mortality [1,2] therefore timely and correct screening means and methods aimed to unveil the underlying potential and acting factors that might lead to their occurrence and progression are of major concern. Along this line, non-invasive techniques for aortic blood flow monitoring, such as the Thoracic Electrical Bio-impedance (Thoracic EBI), are favored because they provide inexpensive yet accurate information on cardiovascular vital parameters, e.g. cardiac output, stroke volume, etc. EBI may be correlated with the Electrocardiographic (ECG) and Respiratory signals, to provide relevant cardio-respiratory indices such as the *cardiac output* (CO), the *stroke volume* (SV), the *flow time* (FT), the *systemic vascular resistance* (SVR), to mention but a few. Two dual electrodes – a pair for electrical current input, and a pair for voltage output – are used to produce the signals. The voltage to current ratio is the EBI – an electrical impedance. EBI is sensitive to the changes in the electric conductivity of the tissues. In particular, two versions of EBI, called *cardio-vascular impedance* (CVI) techniques – the *impedance cardiography* (ICG) and the *electro-cardiometry* (ECM), Fig. 1 – have been introduced with the intention to monitor the cardio-vascular activity as perceived through the changes in the electrical conductivity of the arterial flow. This paper introduces a mathematical model for CVI technique aimed to elicit EBI cardio-vascular indices at the brachial (arm) level, Fig. 2, its numerical implementation, and numerical simulation results.

2. THE BRACHIAL IMPLEMENTATION OF CVI

CVI is sensitive mainly to the pulsatile variations of the arterial flow. Two dual electrodes – for electrical current input, and for voltage output – are used to acquire suitable data to compute the EBI signal. EBI time derivative senses the dynamics of the blood electro-conductivity, which varies during the cardiac cycle due to the complex (re)orientation of the red blood cells.

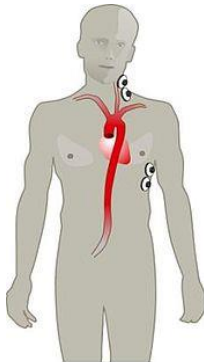


Fig. 1. The Thoracic EBI – ECM implementation

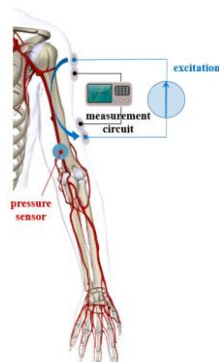


Fig. 2. CVI brachial implementation

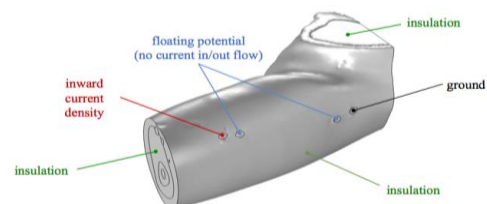


Fig. 3. The boundary conditions

The time derivative senses the changes in the electrical conductivity of blood during the cardiac cycle, due to the complex blood flow induced orientation of the red blood cells.

3. THE MATHEMATICAL MODEL OF THE CVI AND ITS NUMERICAL SOLUTION

The mathematical model of the electric field for steady state conditions is described by the elliptic partial differential equation

$$\nabla \cdot (\sigma \nabla V) = 0, \tag{1}$$

where, V [V] is the electric potential.

The electric current density, \mathbf{J} [A/m²], inside the electro-conductive biological volume (here, the arm) is given by $\mathbf{J} = -\sigma \nabla V$, where σ [S/m] is the electrical conductivity.

The boundary conditions that close the electric field problem are presented in Fig. 3. The mathematical model is solved numerically, by using the finite element method (FEM) as implemented by [3].

The computational domain is presented in Fig. 4.a, while Fig. 4.b depicts the FEM mesh [4]. It was built using reconstruction techniques out of medical CT/MRI scans [5]. Details of this approach, concerning the ECM study, may be found in [6].

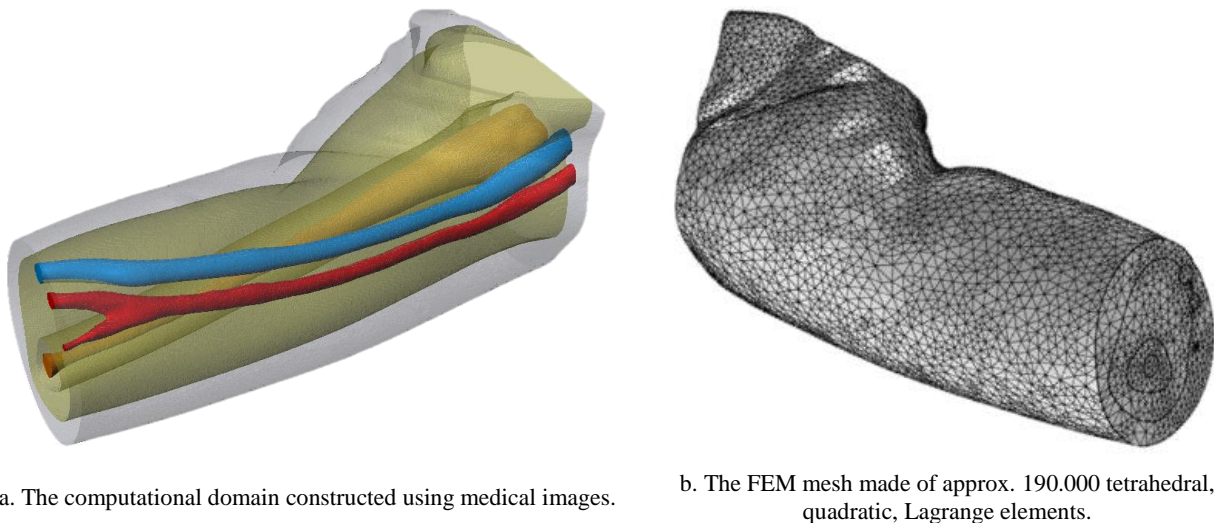


Fig. 4. The forearm as the computational domain

Table I lists the electrical conductivity, σ [S/m], for the main anatomic subdomains accounted for in this study [6]; we also assume that the vascular blood flow is stationary.

Table I. The electrical conductivity of the different anatomic regions

Tissue	Electrical Conductivity [S/m]
Blood	0.66
Bone	0.006
Muscle	0.355
Marrow	0.00247
Arm	0.17

A special case is that of the electrical conductivity of the arterial blood. Previous studies [7-8] provide for an equivalent conductivity for the blood, assumed to be a dilute suspension of ellipsoidal particles (RBCs) in plasma.

Maxwell-Fricke equation is used to define the conductivity of a suspension of RBCs in an electrical field. As later shown [6], the solution to the CVI may be divided into three steps. First, the pulsatile arterial (brachial) blood flow is solved.

Then, assuming that the flow is confined to an equivalent volume delimited by two cross-sectional interfaces (inlet and outlet), the (dynamic) electrical conductivity of the blood is derived. Finally, the electrical field problem is solved.

4. NUMERICAL SIMULATION RESULTS AND DISCUSSION

Fig. 5 gives the electrical conductivity of the brachial blood, evaluated as shown in Sect. 3.

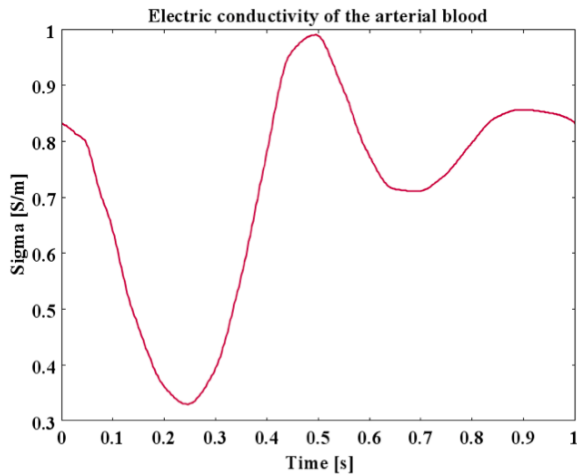


Fig. 5. The electrical conductivity of the brachial blood.

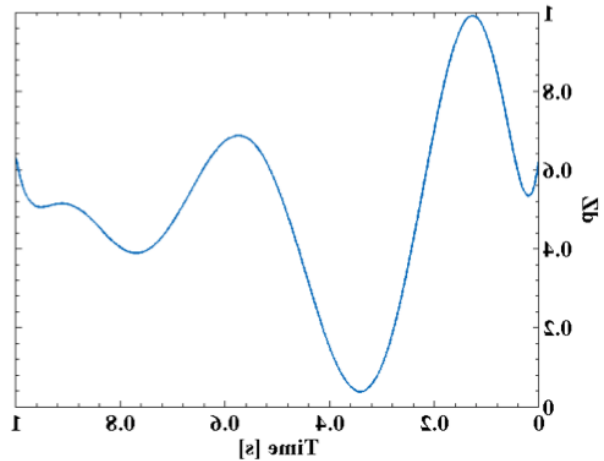


Fig. 6. The derived bioimpedance of the arm, normalized by its peak value s .

Details of this approach, concerning the ECM study, may be found in [9]. Fig. 5 shows the CVI of the arm computed using the properties in Table 1 and Fig. 5.

Apparently, CVI dynamics follows closely the variation of the electrical conductivity of the blood in the brachial artery.

This is a new result as such reported studies are concerned with aortic blood flow dynamics. The usage of anatomically realistic computational domain to compute arterial flows may pose concerns in evaluating the pending electrical conductivity.

To circumvent this difficulty we propose an equivalent conductivity obtainable out of analytic results by using averaging techniques.

Several effects (e.g., the uneven change in conductivity for accelerating and decelerating flow) may not be evidenced in this way, but the sensitivity of CVI to the arterial flow dynamics is clearly perceivable.

The work reported here addresses the direct problem of CVI, used to extract blood flow parameters. Its solution opens the path to the inverse CVI problem, aimed to unveil the flow dynamics out of CVI experimental data. This makes the object of a future research.

5. CONCLUSIONS

The impedance signal at arm level may follow closely the brachial velocity profile therefore the CVI measured on the upper limb is relevant for the assessment of cardiovascular parameters.

Numerical simulations may be conducted on more realistic anatomical structures (brachial artery and vein, bone, marrow, and muscle tissue) constructed using medical image datasets.

The “peristaltic” hemodynamic flow characterization may be of interest for the brachial and superficial veins too.

Electrical conductivity models for the venous blood flow are needed.

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A Nonlinear Control Law For A Hexacopter Vehicle

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Abstract: The paper focuses on the control of a hexacopter, the main purpose being to design a control law able to guarantee robust stability performances with respect to the modeling uncertainties. The hexacopter has several characteristics that give it operational advantages over other types of flying unmanned vehicles. One of these is that in the hexacopter can be stabilized easier. Numerical simulations test the stability properties when sliding mode control laws are applied on the non-linear dynamic model of the hexacopter

Key Words: hexacopter, sliding mode control, autonomous UAV, nonlinear control

I. INTRODUCTION

Unmanned autonomous aerial vehicle (UAV) have become a great area of interest in the last years. The present paper focuses on the control of a hexacopter which is a unmanned aerial vehicle with six rotors, the main purpose being to design a control law able to guarantee robust stability performances in the presence of modeling uncertainties.

The hexacopter consists of six arms all connected symmetrically to the central hub. A propeller is attached at the end of each arm.

The considered hexacopter has all the propellers with fixed pitch blades, meaning that propellers can not be tilted.

A hexarotor has been chosen because of the advantages it presents (high agility and maneuverability, easy stability in case of external perturbation, such as wind, payload redundancy in the case of a propeller failure).

II. MATHEMATICAL MODELLING OF THE HEXACOPTER’S DYNAMICS AND KINEMATICS

The total system model will be obtain as

$$\left\{ \begin{array}{l} \ddot{x} = -\frac{k_{fx}}{m}\dot{x} + \frac{u_x u_1}{m}, \quad \ddot{\phi} = \frac{1}{I_x} [\dot{\theta}\dot{\psi}(I_y - I_z) - K_{fax}\dot{\phi}^2 - J_r\Omega_r\dot{\theta} + u_2] \\ \ddot{y} = -\frac{k_{fy}}{m}\dot{y} + \frac{u_y u_1}{m}, \quad \ddot{\theta} = \frac{1}{I_y} [\dot{\phi}\dot{\psi}(I_z - I_x) - K_{fay}\dot{\theta}^2 + J_r\Omega_r\dot{\phi} + u_3] \\ \ddot{z} = -\frac{k_{fz}}{m}\dot{z} + \frac{u_1 \cos\phi \cos\theta}{m} - g, \quad \ddot{\psi} = \frac{1}{I_z} [\dot{\phi}\dot{\theta}(I_x - I_y) - K_{faz}\dot{\psi}^2 + u_4] \end{array} \right.$$

where $u_x = \cos\phi \cos\psi \sin\theta + \sin\phi \sin\psi$, $u_y = \cos\phi \sin\psi \sin\theta - \sin\phi \cos\psi$ and x , y and z denote the position of the vehicle with respect to the Earth fixed frame, ϕ is roll angle, θ is pitch angle and ψ denotes the yaw angle.

The hexacopter’s total thrust force and torque control inputs u_1, u_2, u_3, u_4 are related to the six motor’s speed by the following equations: $U^T = [u_1 \ u_2 \ u_3 \ u_4]$ is the vector of artificial input variables [5]:

$$\begin{bmatrix} u_4 \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} b & b & b & b & b & b \\ -\frac{b\ell}{2} & -b\ell & -\frac{b\ell}{2} & \frac{b\ell}{2} & b\ell & \frac{b\ell}{2} \\ -b\ell\frac{\sqrt{3}}{2} & 0 & b\ell\frac{\sqrt{3}}{2} & b\ell\frac{\sqrt{3}}{2} & 0 & -b\ell\frac{\sqrt{3}}{2} \\ -d & d & -d & d & -d & d \end{bmatrix} \begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \\ \Omega_5^2 \\ \Omega_6^2 \end{bmatrix}$$

Where b is thrust coefficient, d is drag coefficient and ℓ is arm length. The rotors velocities can be derived from the above equations resulting:

$$\begin{cases} \Omega_1^2 = \frac{1}{6b\ell}(\ell u_4 + 2u_1 - \frac{b\ell}{d}u_3), & \Omega_4^2 = \frac{1}{6b\ell}(\ell u_4 - 2u_1 + \frac{b\ell}{d}u_3) \\ \Omega_2^2 = \frac{1}{6b\ell}(\ell u_4 + u_1 - \sqrt{3}u_2 + \frac{b\ell}{d}u_3), & \Omega_5^2 = \frac{1}{6b\ell}(\ell u_4 - u_1 + \sqrt{3}u_2 - \frac{b\ell}{d}u_3) \\ \Omega_3^2 = \frac{1}{6b\ell}(\ell u_4 - u_1 - \sqrt{3}u_2 - \frac{b\ell}{d}u_3), & \Omega_6^2 = \frac{1}{6b\ell}(\ell u_4 + u_1 + \sqrt{3}u_2 + \frac{b\ell}{d}u_3) \end{cases}$$

III. A SLIDING MODE BASED DESIGN METHODOLOGY FOR THE CONTROL LAW

Sliding mode control is an efficient nonlinear design method allowing to provide stability and tracking performance under model uncertainties conditions of the controlled plant [6].

The following problems for the hexacopter control have been considered.

a. Position Control on the x and y Axes

- Position Control on the x Axis

The final expression of the control input becomes: $u_x = \frac{m}{U_4} [\ddot{x}_{1d} - \alpha_1(x_2 - \dot{x}_{1d}) - k_1 \text{sat}(s_1) - \ell_1 s_1]$

- Position Control on the y Axis

Considering that the the algorithm is similar to the position control on the x axis, the final form of the control input is:

$$u_y = \frac{m}{U_4} [\ddot{x}_{3d} - \alpha_2(x_4 - \dot{x}_{3d}) - k_2 \text{sat}(s_2) - \ell_2 s_2]$$

b. Altitude Control

The equation describing the vertical motion of the hexacopter is: $\dot{x}_6 = \cos x_7 \cos x_9 \frac{1}{m} u_4 - g$.

Proceeding as in the previous case, the control law for avoiding chatter is:

$$u_4 = \frac{m}{\cos x_7 \cos x_9} [g + \ddot{x}_{5d} - \alpha_3(x_6 - \dot{x}_{5d}) - k_3 \text{sat}(s_3) - \ell_3 s_3].$$

- Roll Control

Considering the roll equation: $\dot{x}_8 = x_{10} x_{12} \left(\frac{I_y - I_z}{I_x} \right) - \frac{J_r}{I_x} x_{10} \Omega + \frac{1}{I_x} u_1$.

Following the same algorithm as the one previously used, the control law for the roll angle has the expression:

$$u_1 = I_x \left[-x_{10} x_{12} \left(\frac{I_y - I_z}{I_x} \right) + \frac{J_r}{I_x} x_{10} \Omega + \ddot{x}_{7d} - \alpha_4(x_8 - \dot{x}_{7d}) - k_4 \text{sat}(s_4) - \ell_4 s_4 \right]$$

- Pitch Control

As in previous cases, the expression for the pitch control law is:

$$u_2 = I_y \left[-x_8 x_{12} \left(\frac{I_z - I_x}{I_y} \right) + \frac{J_r}{I_y} x_8 \Omega + \ddot{x}_{9d} - \alpha_5 (x_{10} - \dot{x}_{9d}) - k_5 \text{sat}(s_5) - \ell_5 s_5 \right]$$

- Yaw Control

Using the steps as in previous cases, in order to avoid the chattering phenomenon, the yaw control law has the following form:

$$u_3 = I_z \left[-x_8 x_{10} \left(\frac{I_x - I_y}{I_z} \right) + \ddot{x}_{11d} - \alpha_6 (x_{12} - \dot{x}_{11d}) - k_6 \text{sat}(s_6) - \ell_6 s_6 \right]$$

All coefficients above $\alpha_i, k_i, \ell_i, i=1...6$, are positive constants chosen such that satisfactory response time is obtained. Moreover, the *saturation* function have been used in these control laws instead of the *signum* function in order to reduce the “chattering” effects. In the above equations, $x_{(2i-1)d}, i=1...6$ denote the desired references signals and the state vector has the elements

$x_1 = x, x_2 = \dot{x}, x_3 = y, x_4 = \dot{y}, x_5 = z, x_6 = \dot{z}, x_7 = f, x_8 = \dot{f}, x_9 = q, x_{10} = \dot{q}, x_{11} = y, x_{12} = \dot{y}$ and the sliding surfaces are given by $s_i = \dot{e}_i + \alpha_i e_i, i=1...6$ with $\alpha_i > 0$ and the tracking error is defined as $e_i = x_{2i-1} - x_{(2i-1)d}, i=1...6$.

IV. NUMERICAL SIMULATIONS

The nominal parameters of the hexacopter are: $\ell = 0,4m$ - arm length, $m = 1,64kg$ - mass, $I_x = 0,044kg \cdot m^2$ - inertia on x axis, $I_y = 0,044kg \cdot m^2$ - inertia on y axis, $I_z = 0,088kg \cdot m^2$ - inertia on z axis, $b = 10 \cdot 10^{-6} N \cdot s^2$ - thrust coefficient, $d = 0,3 \cdot 10^{-6} N \cdot m \cdot s^2$ - drag coefficient and $J_r = 90 \cdot 10^{-6} N \cdot m^2$ - rotor inertia.

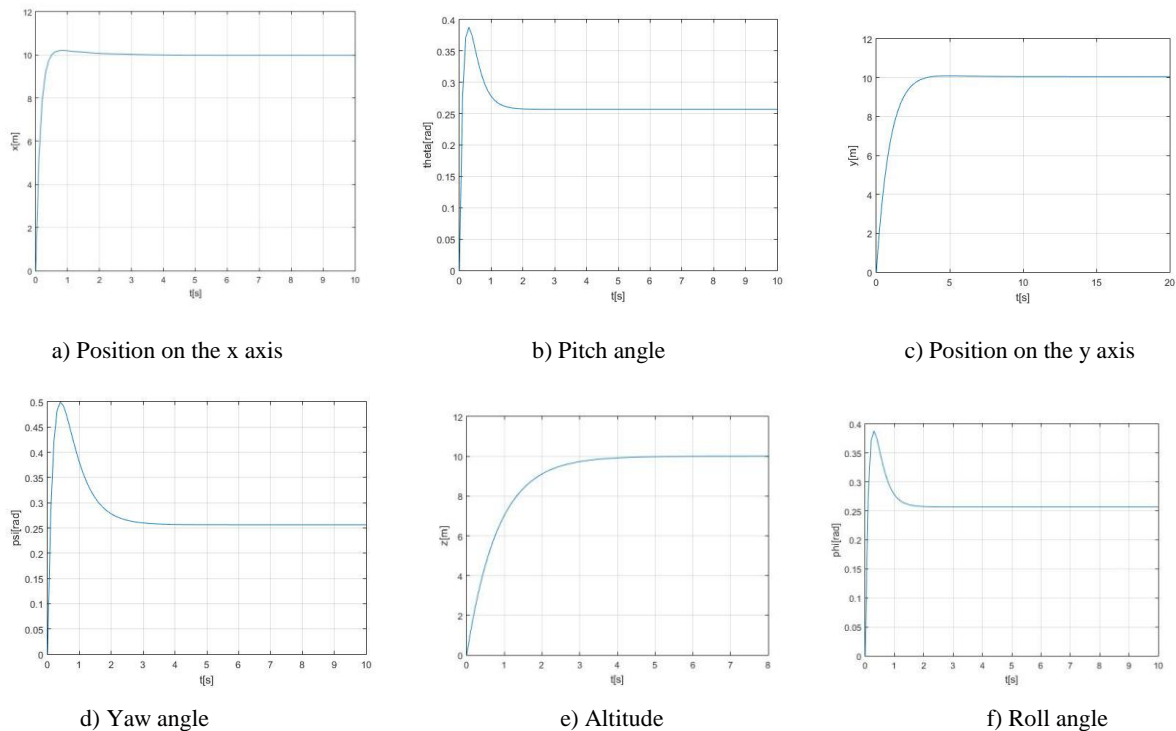


Figure 1. Time responses

The vehicle starts from (0,0,0) and it has to travel to the position defined by (10,10,10). The desired values for pitch angle, yaw angle and, respectively, roll angle are 15 degrees.

The hexacopter reaches the desired position around $t = 4s$. It also achieves the desired angle values in less than 3s (Figure 1).

The numerical results and simulations indicate that the sliding mode controllers provide good stability and tracking performances.

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